NAME _____ SECTION _____

MIDTERM II

Math 124, Section G

November 22, 2005

Problem	Total Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	
6(Bonus)	2	

- No book, notes or graphing calculators are allowed. You may use a scientific calculator.

- Show all your work to get full credit.

- Read instructions for each problem CAREFULLY.

- Check your work!

1. (20pts) Find the following derivatives. You do not have to simplify.

   (a) $f(x) = \sqrt[5]{\frac{\sec(x)\cdot 2^{x^2}}{1-x^4}}$. Using logariphmic differentiation, find $\frac{f'(x)}{f(x)}$.

   Answer. $\frac{f'}{f} = (\ln(f))' = \frac{1}{5}(\tan(x) + 2x\ln 2 + \frac{4x^3}{1-x^4})$

   (b) $f(x) = \tan(\sin(x)) + \cos(\cot(x))$.

   Answer. $f(x)' = \sec^2(\sin x)\cos x + \sin(\cot x)\csc^2 x$

   (c) $f(x) = (\ln x)^{\cos x}$

   Answer. $f(x)' = (\ln x)^{\cos x}(\frac{\cos x}{x\ln x} - \sin x \ln\ln x)$

   (d) $y = arcsin(t)$

   Answer. $\frac{d^2y}{dt^2} = \frac{t}{\sqrt{(1-t^2)^3}}$

2. (20pts) A round ballon of radius 10cm has been punctured and is shrinking uniformly (i.e. keeping the shape of a sphere but just getting smaller). Estimate by linear approximation the change of the volume $V = \frac{4}{3}\pi r^3$ of the baloon, when its radius is decreased from 10 cm to 9.8 cm. Is your estimate bigger or smaller than the actual value? Explain without computing the actual volume at r = 9.8 cm.

   Answer.
   $V'(r) = 4\pi r^2$
   $V'(10) = 400\pi$
   The equation of the tangent line to $V(r)$ at $r = 10$:

   $$y - V(10) = 400\pi(x - 10)$$

   To estimate change in volume when r changes from 10 to 9.8, plug in $r = 9.8$:

   $$y - V(10) = 400\pi(-0.2) = -80\pi \text{ cm}^3$$

3. (20pts) Consider the curve given by the equation

   $$\frac{1}{x+1} + \frac{1}{y+1} = 1$$

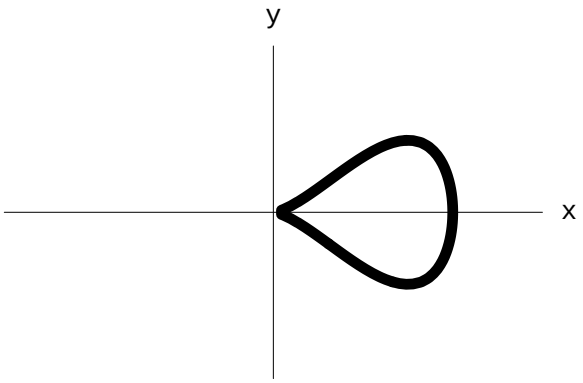
   (a) Find the equation of the tangent line to the curve at the point $(1, 1)$.

   Answer. $y = 2 - x$

(b) Using linear approximation estimate y when $x = 0.9$

Answer. $y \simeq 1.1$

4. (20pts) The graph of the equation $y^2 = x^3(2-x)$ is given below:



(a) Use implicit differentiation to find $\frac{dy}{dx}$.

Answer. $\frac{dy}{dx} = \frac{x^2(3-2x)}{y}$

(b) Find all points on the graph where the tangent line is horizontal. Make sure to give both coordinates of any such points. Leave your answer in EXACT form.

Answer: $(\frac{3}{2}, \frac{3\sqrt{3}}{4})$, $(\frac{3}{2}, -\frac{3\sqrt{3}}{4})$

(c) Find all points on the graph where the tangent line is vertical.

Answer. $(2, 0)$.

5. (20pts) An observer on the ground sights an approaching plane flying at constant speed and at an altitude of 20000ft. From his point of view, the plane's angle of elevation is increasing at 0.5^o per second when the angle is 60^o. What is the speed of the plane? Note that speed is the absolute value of the velocity and therefore must be positive.

Answer. Let x be the distance from the plane to the point on its course which is right above the observer. Let θ the angle of elevation of the plane, measured in radians, Then

$$x = 20000 \cot\theta$$

Therefore,

$$\frac{dx}{dt} = -20000 \csc^2(\theta)\frac{d\theta}{dt}$$

Plug in $\theta = \pi/3$, $\frac{d\theta}{dt} = 0.5^o = \pi/360$, $\csc(\pi/3) = 2/\sqrt{3}$

$$\frac{dx}{dt} = -20000\frac{4}{3}\frac{\pi}{360} = -\frac{2000}{27}\pi \simeq -74\pi$$

Answer: $74\pi \simeq 232.7$ ft/s.