MIDTERM I
Math 124, Section G
October 25, 2005

Problem	Total Points	Score
1	30	
2	10	
3	20	
4	20	
5	20	
Total	100	
6 (bonus)	2	
7 (bonus)	2	
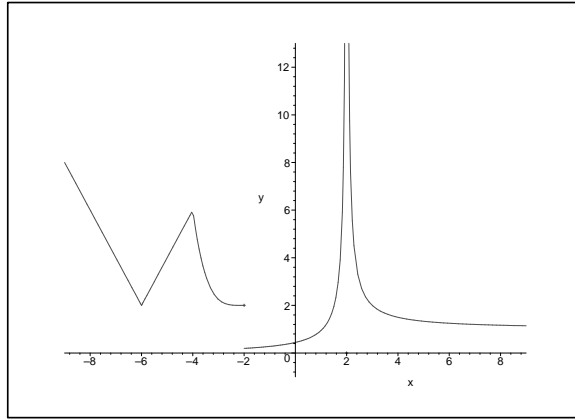
1. (30pts) Compute the following limits:

   (a) $\lim\limits_{x \to -1} \frac{x^2+2x+1}{x+1}$

   Answer. 0

   (b) $\lim\limits_{x \to 0} \frac{\sin^2 x}{x^2}$

   Answer. 1

   (c) $\lim\limits_{x \to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

   Answer. 1

2. (10pts) The graph of the function f is given above.

   (a) List all points on the interval $[-7, 7]$ where f is NOT continuous

   Answer. $x = -2$, $x = 2$

   (b) List all points on the interval (-7,7) where f is NOT differentiable

   Answer. $x = -6$, $x = -4$, $x = -2$, $x = 2$

3. (20pts) Find the equation of the tangent line to the graph of $f(x) = \sqrt{x+1}$ at the point $x = 3$.
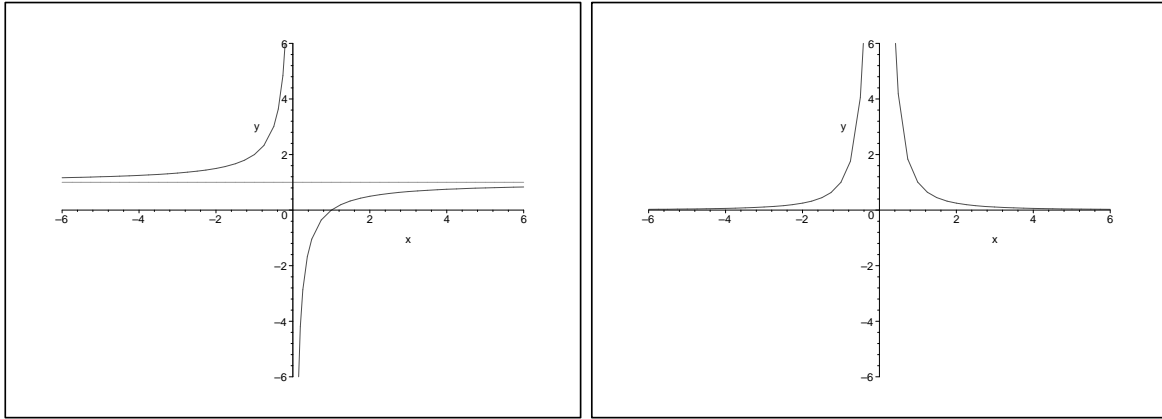
   Note: You must use limits when computing the slope: no credit for using shortcuts from chapter 3 or from your previous experience with calculus.

   Answer. $y - 2 = \frac{1}{4}(x - 3)$

4. (20pts) Let $f(x) = \frac{x-1}{x}$

   (a) Find horizontal asymptotes of $f(x)$ if they exist.

   Answer. $y = 1$

2

(b) Graph $f(x)$. Label horizontal asymptotes on the graph. (Hint: use that $\frac{x-1}{x} = 1 - \frac{1}{x}$)

Answer. See the left graph above.

(c) What are the domain and range of $f(x)$?

Answer: Domain $\{x \neq 0\}$, Range $\{y \neq 1\}$

(d) Where is f a continuous function?

Answer. $\{x \neq 0\}$

(e) Where is f a differentiable function?

Answer. $\{x \neq 0\}$

(f) Compute $f'(x)$ **using limits**.

Answer. $f'(x) = \frac{1}{x^2}$

(g) Graph $f'(x)$, give the domain and range of $f'(x)$.

Answer. See the right graph above. Domain $\{x \neq 0\}$, Range $\{y > 0\}$.

5. (20pts)   The percentage of the students at the University of Exponential Grade Inflation who get an A for their calculus class is claimed to grow exponentially. In other words, it is modeled by the function

$$f(t) = Ce^{rt}$$

where C, r are positive numbers. If 5% of all students got an A in the year 1970 when the University offered its first calculus course, and reached 20% in 2005, estimate when all students will be getting A for calculus.

Note: You may leave your answer in the exact form.

Answer. $f(t) = 5e^{\frac{\ln 4}{35}t} \simeq 5e^{0.04t}$. $f(t)$ reaches 100% when $t = \frac{35 \ln 20}{\ln 4} \simeq 75$ years. Thus, everybody will get an A in 2045.