Some of the following problems were taken from midterms given by other professors in previous years (prior to Autumn 2005). Others are problems I've made up. I've selected these problems specifically because they are good examples of the types of problems you need to be able to do for the midterm.

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1. Compute the derivative of each the following functions (do not simplify your answer):
   (a) $f(x) = \cos(e^{-x^2})$
   (b) $g(x) = \dfrac{xe^x}{\sqrt{1-x^2}}$
   (c) $y = x^{\ln(x)}\sec(x)$
   (d) $h(t) = \tan^{-1}(\sqrt{2^t+1})$
   (e) $s(t) = \sin(at)^{\tan(t)}$, where $a$ as a constant
   (f) $\phi(x) = \sin^{-1}(x\sin(2x))$
   (g) $\psi(x) = \tan^{-1}(\ln(3x^2+x))$
2. Consider the curve whose graph is given by the equation $\sin(x)y + \ln(y) = xy$.
   (a) Verify that $(0, 1)$ is on the curve. Then find the equation of the tangent line to the curve at $(0, 1)$.
   (b) Find $\frac{d^2y}{dx^2}$ at $(0, 1)$. *Hint*: use something you know from part (a).
3. Consider the curve whose graph is given by the equation $5y^3+yx^2+x^2 = 4$. Find the equations of the tangent lines at the $x$-intercepts.
4. The point $(2, 1)$ lies on the curve $x^2\cos(\pi y) + 2y + x = 0$. Find the equation of the tangent line to the curve at the point $(2, 1)$.
5. A particle moves along the $x$-axis so that its position at time $t$ is given by
   $$x(t) = -t^3 + 9t^2 - 23t + 15,$$
   where $t$ is measured in seconds and $x$ is measured in feet.
   (a) Given that the particle reaches $x = 0$ at $t = 1$ sec, find the velocity of the particle at **ALL** times when the particle reaches $x = 0$.
   (b) Find when the velocity of the particle is zero.
   (c) Find when the velocity is maximum.
   (d) Find when the acceleration of the particle is zero.
6. Recall that a baseball diamond is a square with side length 90 feet. At time $t = 0$, a batter starts from home base and runs around the bases at a speed of 18 feet per second. Calculate the rate at which the batter's distance to third base is changing with respect to time when $t = 7.5$ seconds. (*Hint*: first think about where the batter is at when $t = 7.5$ seconds.)

7. A flashlight is laying on the ground 30 feet from a wall. It is on and pointed straight at the wall. Isobel is walking straight from the wall to the flashlight at a constant speed of 3 ft/sec. She is 6 feet tall. How fast is the length of her shadow increasing when she is 18 feet away from the flashlight.

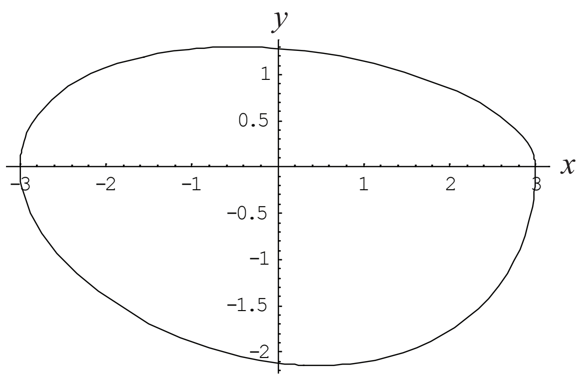
8. A particle in the first quadrant moves along the curve $y = x^3 - 3x$ in such a way that the $x$-coordinate of its position $P$ increases at a steady rate of 7 cm/sec. Let $\ell$ be the line joining $P$ to the origin. How fast is the angle of inclination of $\ell$ changing when $x = 2$ cm? (The angle of inclination of a line is the angle it makes with the positive $x$-axis.)

9. Let $c$ be a constant. Show that the sum of the $x$- and $y$-intercepts of any tangent to the graph of $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is $c$.

10. An object is moving along a path that is given by the following parametric equations:

$$x(t) = 3\cos(\pi t)$$
$$y(t) = \sqrt{e^t}\sin(\pi t)$$

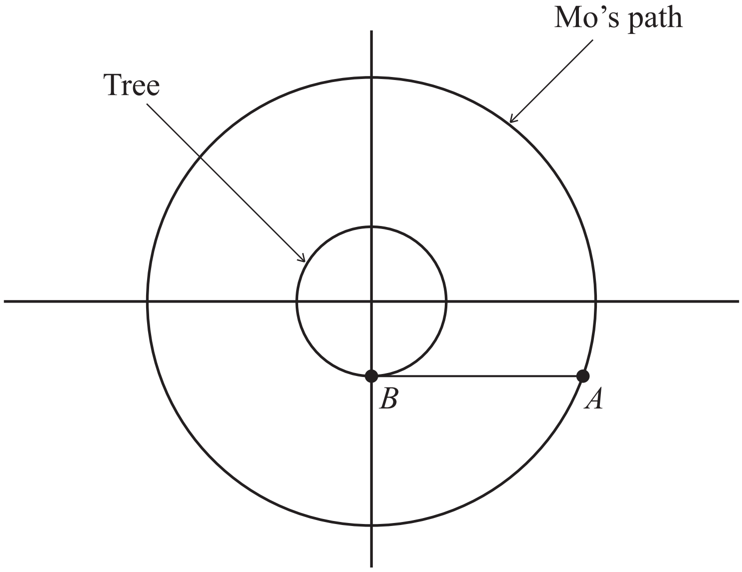
where $0 \le t \le 2$ is measured in seconds and $x$ and $y$ are measured in meters.




(a) Find the horizontal and vertical velocities of the object. Include units.
(b) Find the slope of the curve as a function of time.
(c) Find the equation of the tangent line to the curve at $t = \frac{3}{2}$ seconds.
(d) Determine when the object will be moving in a vertical direction. Find where the object is located at these times and indicate these points on the curve.
(e) Find the speed of the object when $t = \frac{3}{2}$ seconds.

11. (This one is for fun.)

Mo has tied a rope around a tree. Assume that the tree trunk is circular with radius $\frac{1}{\sqrt{\pi}}$ feet. After tieing the rope to the tree, there is 15 feet of rope left. Mo grabs the rope and walks counterclockwise around the tree maintaining a distance of 3.5 feet from the tree and keeping the rope tight (see the diagram below). So the rope is being wrapped around the tree as Mo walks around the tree. It takes Mo 4 seconds to walk around the tree.




(a) Assume that Mo starts from point $A$ as shown in the diagram above. $B$ is the point where the rope is leaving the tree (i.e., the point where the rope is tangent to the tree). Find the coordinates of $A$.

(b) As Mo moves around the tree, the point $B$ will be moving around the tree as well. Let $x(t)$ and $y(t)$ represent the location of $B$ at time $t$. Find equations for $x(t)$ and $y(t)$. (*Hint*: first think about how fast this point is moving around the tree and don't forget about where $B$ is when $t = 0$.)

(c) Determine how long it will take before Mo reaches the end of the rope. (*Hint*: don't forget that Mo **always** stays 3.5 feet away from the tree.)

(d) Find the equation of the tangent line that describes the position of the rope at the time when Mo reaches the end of the rope.