Some of the following problems were taken from midterms given by other professors in previous years (prior to Autumn 2005). Others are problems I've made up. I've selected these problems specifically because they are good examples of the types of problems you need to be able to do for the midterm.

- All derivatives must be calculated using limits (no differentiation formulas from Chapter 3 or from previous experience with Calculus).
- Show all your work and leave all answers in exact form (unless otherwise stated in the problem).
- Make sure you can do these problems using only a scientific calculator. No graphing calculators will be allowed on the midterm.

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1. Compute the following limits, if they exist. If not, explain why.

   a) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$

   b) $\lim_{x\to 2} \frac{x-1}{x^2-4x+4}$

   c) $\lim_{x\to 0} \frac{\tan(x)}{x}$

   d) $\lim_{x\to 3} \frac{x^2-9}{\sqrt{x}-\sqrt{3}}$

   e) $\lim_{x\to 9} \frac{9-x}{3-\sqrt{x}}$

   f) $\lim_{x\to 2^-} \frac{5x^2-8x-4}{|x-2|}$

   g) $\lim_{x\to \infty} \frac{x^2-1}{x^3+6x+1}$

   h) $\lim_{x\to -\infty} \frac{2x+\sqrt{x^2-x+1}}{3x+1}$

2. A petri dish starts out with a colony of 1000 bacteria, which grows exponentially. Three days later, there are 2000 bacteria in the colony. In how many *more* days will there be 9000 bacteria in the colony?

3. A particle starts at the point $(1,1)$ and heads in a straight line toward the point $(10,13)$ at the constant speed of 5 units per second. Find a formula for its position at time t seconds.

4. Find the equation of the tangent line to the graph of $f(x) = \frac{x}{x+8}$ at the point $(-7,-7)$.

5. Find the unique positive integer n such that the tangent line to the graph of $g(x) = x^n$ has slope 4 at the point $(1,1)$.

6. Is $h(x) = \frac{\sqrt{2-x}}{\sqrt{x+1}}$ continuous at $x = -1$? Is $h(x)$ a continuous function? Explain your answers.

7. Find all points in the domain of the following function where the function is discontinuous:

$$f(x) = \begin{cases} x+1 & \text{if } x \le 1 \\ \frac{1}{x} & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \ge 3 \end{cases}$$

8. Sketch the graph of $y = 3\sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) + 2$ for $0 \le x \le 5$.

9. An object oscillates horizontally at the end of a spring. Its sinusoidal horizontal motion goes from a minimum position of $s = 0$ m to a maximum position of $s = 10$ m. It is at its maximum position at times $t = 4$ sec, $t = 12$ sec, $t = 20$ sec, etc. Find a formula for s as a function of t.

10. The position of a particle moving along a straight line is given by

$$s(t) = \frac{1}{t+2}$$

   (a) Find an expression for the average velocity in the time interval from time t to time $t+h$. Simplify the expression as much as possible.
   (b) Use the simplified expression above to find the instantaneous velocity of the particle at time t.

11. Let a be a fixed real number and $y = x^2$.

   (a) Find an expression for the slope of the secant line to the curve $y = x^2$ going through the points (a, a^2) and $(a+h, (a+h)^2)$. Simplify this expression and take the appropriate limit to find the slope of the curve $y = x^2$ at the point (a, a^2).
   (b) Find the equation of the tangent line to the curve $y = x^2$ at the point (a, a^2).
   (c) For what value(s) of a does this tangent line go through the point $(3, 8)$?

12. Find the derivative of each of the following functions.

   (a) $f(x) = 13x - 8$
   (b) $g(x) = \sqrt{1+2x}$
   (c) $r(t) = \dfrac{3+t}{1-3t}$
   (d) $s(y) = (x^2 + 3x - 1)(x^2 + 4)$
   (e) $m(x) = 2^x$

13. Consider the following limit:

$$\lim_{h \to 0} \frac{\sqrt{a+h-1} - \sqrt{a-1}}{h}.$$

Given that this limit is the derivative of some function $f(x)$ at $x = a$, what is the rule for $f(x)$?

14. If an arrow is shot straight upward on the moon with an initial velocity of 58 m/s, its height (in meters) after t seconds is given by $H(t) = 58t - 0.83t^2$.

   (a) Find the velocity of the arrow when $t = a$.
   (b) When will the arrow hit the moon?
   (c) With what velocity will the arrow hit the moon?

15. Find the equation of the the line tangent to $f(x) = 2x^4 - 3x + 1$ at the point where $x = 1$.

16. Suppose that $g(x) = x^2 + ax + 2$ for some constant a and that the equation of the line tangent to $g(x)$ at $x = 2$ is $y = 8x - 2$. Find a.

17. Let $f(x) = x^2 - 2x - 15$. Find a so that the tangent line at $x = a$ is perpendicular to the tangent line at $x = -3$.

18. Let $g(x) = x^3 - 2x^2 - 5x + 6$. Find a so that the tangent line at $x = a$ goes through the point $(3, -9)$.