Compute $\frac{dy}{dx}$ in each of the following:

1. $y = \sqrt{x^2+1}$

$$y' = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

2. $y = (x^3-1)\sqrt{3x^2+4}$

$$y' = 3x^2\sqrt{3x^2+4} + \frac{(x^3-1)(6x)}{2\sqrt{3x^2+4}}$$

3. $y = (x^2+1)^3(x^3-1)^2$

$$y' = 3(x^2+1)^2(2x)(x^3-1)^2 + 2(x^3-1)(3x^2)(x^2+1)^3$$

4. $y = \dfrac{x^2+1}{1-3x}$

$$y' = \frac{(1-3x)(2x) - (x^2+1)(-3)}{(1-3x)^2}$$

5. $y^3 + x^2y^4 + x^3 = 1$

$$3y^2y' + 2xy^4 + 4x^2y^3y' + 3x^2 = 0$$
$$y' = \frac{-3x^2 - 2xy^4}{3y^2 + 4x^2y^3}$$

6. $y = \cos^{-1}\left(\dfrac{\tan(2x)}{2x}\right)$

$$y' = -\frac{1}{\sqrt{1-\left(\dfrac{\tan(2x)}{2x}\right)^2}} \cdot \frac{4x\sec^2(2x) - 2\tan(2x)}{4x^2}$$

7. $y = \dfrac{x^3}{\sqrt[3]{3x^2-1}}$

$$y' = \frac{3x^2\sqrt[3]{3x^2-1} - \dfrac{x^3(6x)}{3(\sqrt[3]{3x^2-1})^2}}{(\sqrt[3]{3x^2-1})^2}$$

8. $y = \dfrac{10^x}{\ln(10x)}$

$$y' = \frac{\ln(10)10^x\ln(10x) - \dfrac{10^x}{x}}{(\ln(10x))^2}$$

9. $y = \dfrac{(x^2+1)\sqrt{x^2-1}}{3x+2}$

$$y' = \frac{\left[2x\sqrt{x^2-1} + \dfrac{(x^2+1)(2x)}{2\sqrt{x^2-1}}\right](3x+2) - 3(x^2+1)\sqrt{x^2-1}}{(3x+2)^2}$$

10. $y = \left(\dfrac{2x+1}{3x-1}\right)^4$

$$y' = 4\left(\frac{2x+1}{3x-1}\right)^3 \cdot \frac{2(3x-1)-3(2x+1)}{(3x-1)^2}$$

11. $y = \sqrt{1 - \dfrac{1}{x^2+1}}$

$$y' = \frac{1}{2\sqrt{1-\dfrac{1}{x^2+1}}} \cdot \frac{2x}{(x^2+1)^2}$$

12. $y = \sqrt{x + \sqrt{x+\sqrt{x}}}$

$$y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}}$$

13. $y = 2x^2 \sin^3(5x^8)$

$$y' = 4x\sin^3(5x^8) + (2x^2)3\sin^2(5x^8)\cos(5x^8)(40x^7)$$

14. $y = (\tan(x) - \cos(3x^2))^4$

$$y' = 4(\tan(x) - \cos(3x^2))^3\left[\sec^2(x) + \sin(3x^2)(6x)\right]$$

15. $y = \sin(\cos(x)) + \sin(x)\cos(x)$

$$y' = \cos(\cos(x))(-\sin(x)) + \cos^2(x) - \sin^2(x)$$

16. $y = x^2 \csc^5(\sqrt{x-1})$

$$y' = 2x\csc^5(\sqrt{x-1}) - 5x^2\csc^4(\sqrt{x-1})\csc(\sqrt{x-1})\cot(\sqrt{x-1}) \cdot \frac{1}{2\sqrt{x-1}}$$

17. $y = \dfrac{\sqrt{1-\cos(2x)}}{\tan(x)}$

$$y' = \frac{\dfrac{2\sin(2x)\tan(x)}{2\sqrt{1-\cos(2x)}} - \sec^2(x)\sqrt{1-\cos(2x)}}{\tan^2(x)}$$

18. $y = \sin^{-1}(x^2)$

$$y' = \frac{2x}{\sqrt{1-x^4}}$$

19. $y = 3 + \dfrac{5}{\sqrt{x}} + 2\sqrt{x} - \dfrac{1}{x\sqrt{x}}$

$$y' = -\frac{5}{2x\sqrt{x}} + \frac{1}{\sqrt{x}} + \frac{3}{2x^2\sqrt{x}}$$

20. $(x+y)^2 - (x-y)^2 = x^4 + y^4$

$$2(x+y)(1+y') - 2(x-y)(1-y') = 4x^3 + 4y^3y'$$
$$y' = \frac{4x^3 - 2(x+y) + 2(x-y)}{2(x+y) + 2(x-y) - 4y^3}$$

21. $y = \dfrac{1}{4x^3 + 5x^2 - 7x + 8}$

$$y' = -\frac{12x^2 + 10x - 7}{(4x^3 + 5x^2 - 7x + 8)^2}$$

22. $y = \sqrt{\sin^{-1}(2x)}$

$$y' = \frac{1}{2\sqrt{\sin^{-1}(2x)}} \cdot \frac{2}{\sqrt{1-4x^2}}$$

23. $y = \cos^{-1}(\sqrt{1-2x})$

$$y' = -\frac{1}{\sqrt{1-(1-2x)}} \cdot \frac{-2}{2\sqrt{1-2x}}$$

24. $y = \sqrt[3]{4-x}$

$$y' = \frac{-1}{3(\sqrt[3]{4-x})^2}$$

25. $y = \tan^{-1}\left(\dfrac{x-3}{1+3x}\right)$

$$y' = \frac{1}{1 + \left(\dfrac{x-3}{1+3x}\right)^2} \cdot \frac{(1+3x) - 3(x-3)}{(1+3x)^2}$$

26. $y = \ln(\tan(x))$

$$y' = \frac{\sec^2(x)}{\tan(x)}$$

27. $y = \ln(x^2 + \ln(x + \ln(x)))$

$$\frac{1}{x^2 + \ln(x + \ln(x))} \cdot \left(2x + \frac{1}{x + \ln(x)} \cdot \left(1 + \frac{1}{x}\right)\right)$$

28. $y = x^{e^x}$

$$\begin{aligned} \ln(y) &= e^x \ln(x) \\ \frac{y'}{y} &= e^x \ln(x) + \frac{e^x}{x} \\ y' &= \left(e^x \ln(x) + \frac{e^x}{x}\right) x^{e^x} \end{aligned}$$

29. $y = (4x^2 - 7)^{2+\sqrt{x^2-5}}$

$$\begin{aligned} \ln(y) &= (2 + \sqrt{x^2 - 5}) \ln(4x^2 - 7) \\ \frac{y'}{y} &= \frac{2x}{2\sqrt{x^2 - 5}} \ln(4x^2 - 7) + \frac{(2 + \sqrt{x^2 - 5})8x}{4x^2 - 7} \\ y' &= \left(\frac{2x}{2\sqrt{x^2 - 5}} \ln(4x^2 - 7) + \frac{(2 + \sqrt{x^2 - 5})8x}{4x^2 - 7}\right) (4x^2 - 7)^{2+\sqrt{x^2-5}} \end{aligned}$$

30. $\ln(x + y) = \tan^{-1}\left(\frac{x}{y}\right)$

$$\begin{aligned} \frac{1 + y'}{x + y} &= \frac{1}{1 + \frac{x^2}{y^2}} \cdot \frac{y - xy'}{y^2} \\ \frac{1}{x + y} + \frac{y'}{x + y} &= \frac{y^2}{x^2 + y^2} \cdot \frac{y - xy'}{y^2} \\ \frac{1}{x + y} + \frac{y'}{x + y} &= \frac{y}{x^2 + y^2} - \frac{xy'}{x^2 + y^2} \\ y' &= \frac{\frac{y}{x^2 + y^2} - \frac{1}{x + y}}{\frac{1}{x + y} + \frac{x}{x^2 + y^2}} \end{aligned}$$

31. $x^2 + xy + y^2 - 3 = 0$

$$2x + y + xy' + 2yy' = 0$$
$$y' = \frac{-2x - y}{x + 2y}$$

32. $x\cos(y) + y\cos(x) = 1$

$$\cos(y) - x\sin(y)y' + y'\cos(x) - y\sin(x) = 0$$
$$y' = \frac{y\sin(x) - \cos(y)}{\cos(x) - x\sin(y)}$$

33. $y = 2x^{\csc(x)}$

$$\ln(y) = \ln(2x^{\csc(x)})$$
$$\ln(y) = \ln(2) + \csc(x)\ln(x)$$
$$\frac{y'}{y} = -\csc(x)\cot(x)\ln(x) + \frac{\csc(x)}{x}$$
$$y' = \left(-\csc(x)\cot(x)\ln(x) + \frac{\csc(x)}{x}\right) 2x^{\csc(x)}$$

34. $y = t^2 + 2$ and $t = \tan(x^2 - x)$

$$\frac{dy}{dt} = 2t$$
$$\frac{dx}{dt} = (2x - 1)\sec^2(x^2 - x)$$
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$
$$= \frac{2t}{(2x - 1)\sec^2(x^2 - x)}$$
$$= \frac{2\tan(x^2 - x)}{(2x - 1)\sec^2(x^2 - x)}$$

35. $y = 3^{3x^2}$

$$y' = \ln(3)3^{3x^2}(6x)$$

36. $y = \ln\left(\dfrac{x-1}{x+1}\right)$

$$y' = \frac{x+1}{x-1} \cdot \frac{(x+1)-(x-1)}{(x+1)^2}$$

37. $y = (x^2+4)^4(x^3-3)^{3/4}$

$$y' = 4(x^2+4)^3(2x)(x^3-3)^{3/4} + \frac{3}{4}(x^3-3)^{-1/4}(3x^2)(x^2+4)^4$$

38. $y = 2\sqrt{4\sin(x) - 6\cos(2x)}$

$$y' = \frac{4\cos(x) + 12\sin(2x)}{\sqrt{4\sin(x) - 6\cos(2x)}}$$

39. $x^2 + \cos(2y) = 3$

$$\begin{aligned} 2x - 2\sin(2y)y' &= 0 \\ y' &= \frac{x}{\sin(2y)} \end{aligned}$$

40. $y = (e^{\sin(x)} - \sqrt{2x})(x^2-9)(\tan(x) - \cot(x))^4$

$$\begin{aligned} y' = &\left((\cos(x)e^{\sin(x)} - \frac{1}{\sqrt{2x}}\right)(x^2-9)(\tan(x)-\cot(x))^4 \\ &+ (e^{\sin(x)} - \sqrt{2x})(2x)(\tan(x)-\cot(x))^4 \\ &+ (e^{\sin(x)} - \sqrt{2x})(x^2-9)4(\tan(x)-\cot(x))^3(\sec^2(x)+\csc^2(x)) \end{aligned}$$