Compute $\frac{dy}{dx}$ in each of the following:

1. $y = \sqrt{x^2+1}$

$$y' = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

2. $y = (x^3-1)\sqrt{3x^2+4}$

$$y' = 3x^2\sqrt{3x^2+4} + \frac{(x^3-1)(6x)}{2\sqrt{3x^2+4}}$$

3. $y = (x^2+1)^3(x^3-1)^2$

$$y' = 3(x^2+1)^2(2x)(x^3-1)^2 + 2(x^3-1)(3x^2)(x^2+1)^3$$

4. $y = \dfrac{x^2+1}{1-3x}$

$$y' = \frac{(1-3x)(2x) - (x^2+1)(-3)}{(1-3x)^2}$$

5. $y = \dfrac{x^3}{\sqrt[3]{3x^2-1}}$

$$y' = \frac{3x^2\sqrt[3]{3x^2-1} - \dfrac{x^3(6x)}{3(\sqrt[3]{3x^2-1})^2}}{(\sqrt[3]{3x^2-1})^2}$$

6. $y = \dfrac{10^x}{\ln(10x)}$

$$y' = \frac{\ln(10)10^x\ln(10x) - \dfrac{10^x}{x}}{(\ln(10x))^2}$$

7. $y = \dfrac{(x^2+1)\sqrt{x^2-1}}{3x+2}$

$$y' = \frac{\left[2x\sqrt{x^2-1} + \dfrac{(x^2+1)(2x)}{2\sqrt{x^2-1}}\right](3x+2) - 3(x^2+1)\sqrt{x^2-1}}{(3x+2)^2}$$

8. $y = \left(\dfrac{2x+1}{3x-1}\right)^4$

$$y' = 4\left(\frac{2x+1}{3x-1}\right)^3 \cdot \frac{2(3x-1) - 3(2x+1)}{(3x-1)^2}$$

9. $y = \sqrt{1 - \dfrac{1}{x^2+1}}$

$$y' = \frac{1}{2\sqrt{1 - \dfrac{1}{x^2+1}}} \cdot \frac{2x}{(x^2+1)^2}$$

10. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$$y' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \frac{1}{2\sqrt{x}}$$

11. $y = 2x^2 \sin^3(5x^8)$

$$y' = 4x \sin^3(5x^8) + (2x^2)3\sin^2(5x^8)\cos(5x^8)(40x^7)$$

12. $y = (\tan(x) - \cos(3x^2))^4$

$$y' = 4(\tan(x) - \cos(3x^2))^3 \left[\sec^2(x) + \sin(3x^2)(6x)\right]$$

13. $y = \sin(\cos(x)) + \sin(x)\cos(x)$

$$y' = \cos(\cos(x))(-\sin(x)) + \cos^2(x) - \sin^2(x)$$

14. $y = x^2 \csc^5(\sqrt{x-1})$

$$y' = 2x\csc^5(\sqrt{x-1}) - 5x^2\csc^4(\sqrt{x-1})\csc(\sqrt{x-1})\cot(\sqrt{x-1}) \cdot \frac{1}{2\sqrt{x-1}}$$

15. $y = \dfrac{\sqrt{1-\cos(2x)}}{\tan(x)}$

$$y' = \frac{\dfrac{2\sin(2x)\tan(x)}{2\sqrt{1-\cos(2x)}} - \sec^2(x)\sqrt{1-\cos(2x)}}{\tan^2(x)}$$

16. $y = 3 + \dfrac{5}{\sqrt{x}} + 2\sqrt{x} - \dfrac{1}{x\sqrt{x}}$

$$y' = -\frac{5}{2x\sqrt{x}} + \frac{1}{\sqrt{x}} + \frac{3}{2x^2\sqrt{x}}$$

17. $y = \dfrac{1}{4x^3 + 5x^2 - 7x + 8}$

$$y' = -\frac{12x^2 + 10x - 7}{(4x^3 + 5x^2 - 7x + 8)^2}$$

18. $y = \sqrt[3]{4-x}$

$$y' = \frac{-1}{3(\sqrt[3]{4-x})^2}$$

19. $y = \ln(\tan(x))$

$$y' = \frac{\sec^2(x)}{\tan(x)}$$

20. $y = \ln(x^2 + \ln(x + \ln(x)))$

$$\frac{1}{x^2 + \ln(x + \ln(x))} \cdot \left(2x + \frac{1}{x + \ln(x)} \cdot \left(1 + \frac{1}{x}\right)\right)$$

21. $y = t^2 + 2$ and $t = \tan(x^2 - x)$

$$\begin{aligned}
\frac{dy}{dt} &= 2t \\
\frac{dx}{dt} &= (2x-1)\sec^2(x^2-x) \\
\frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \\
&= \frac{2t}{(2x-1)\sec^2(x^2-x)} \\
&= \frac{2\tan(x^2-x)}{(2x-1)\sec^2(x^2-x)}
\end{aligned}$$

22. $y = 3^{3x^2}$

$$y' = \ln(3)3^{3x^2}(6x)$$

23. $y = \ln\left(\dfrac{x-1}{x+1}\right)$

$$y' = \frac{x+1}{x-1} \cdot \frac{(x+1)-(x-1)}{(x+1)^2}$$

24. $y = (x^2+4)^4(x^3-3)^{3/4}$

$$y' = 4(x^2+4)^3(2x)(x^3-3)^{3/4} + \frac{3}{4}(x^3-3)^{-1/4}(3x^2)(x^2+4)^4$$

25. $y = 2\sqrt{4\sin(x) - 6\cos(2x)}$

$$y' = \frac{4\cos(x) + 12\sin(2x)}{\sqrt{4\sin(x) - 6\cos(2x)}}$$

26. $y = (e^{\sin(x)} - \sqrt{2x})(x^2-9)(\tan(x) - \cot(x))^4$

$$\begin{aligned} y' = &\left((\cos(x)e^{\sin(x)} - \frac{1}{\sqrt{2x}}\right)(x^2-9)(\tan(x)-\cot(x))^4 \\ &+ (e^{\sin(x)} - \sqrt{2x})(2x)(\tan(x)-\cot(x))^4 \\ &+ (e^{\sin(x)} - \sqrt{2x})(x^2-9)4(\tan(x)-\cot(x))^3(\sec^2(x)+\csc^2(x)) \end{aligned}$$