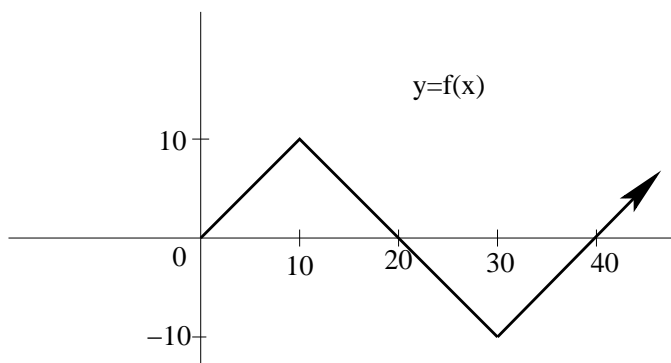


Practice Midterm I
 Math 124, Sections C, D
 April, 2006

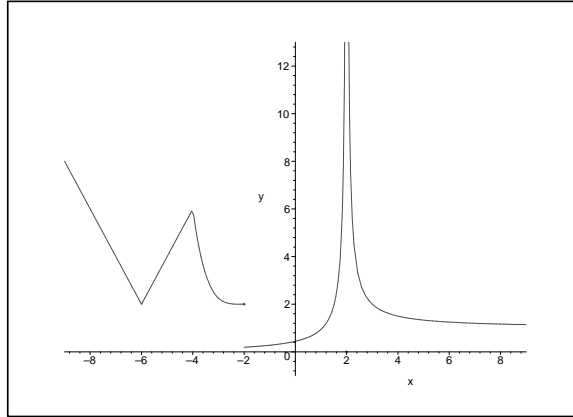
Note: you must use limits to find derivatives and slopes of tangent lines. No credit will be given for using shortcuts you might have learned outside of the classroom (not to worry: you'll have plenty of opportunity to use these shortcuts later in the course).

Note: the actual midterm is “no notes, books or graphing calculators”. You may use your “simple” scientific calculator on the exam. You have to show ALL YOUR WORK on the exam to get credit.

1. Here is the picture of the graph of a distance function $y = f(x)$. Distance is in feet, time is in seconds.



- (a) $f'(7) =$
- (b) $\lim_{x \rightarrow 0} f(x + 20) =$
- (c) $\lim_{x \rightarrow 0} \frac{f(x + 20)}{x} =$
- (d) $\lim_{x \rightarrow 0} \frac{f(x + 20) - 20}{x - 20} =$
- (e) The average velocity on the time interval $[0,40] =$
- (f) The maximum velocity on the time interval $[0,40] =$
- (g) Let $g(x) = \frac{x}{x+1}$ and $h(x) = f(g(x))$. Find $\lim_{t \rightarrow 10} h(t)$



2. The picture above is the graph of a function $y = f(x)$. Determine at which points the function is discontinuous.

3. Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$

(b) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{x}$

(c) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

(d) $\lim_{t \rightarrow 3} \frac{t^3 - 9t}{t^2 - 9}$

(e) $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta^2}$

(f) $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta^3}$

(g) $\lim_{h \rightarrow 2} \frac{\frac{1}{h} - \frac{1}{h-2}}{h-2}$

(h) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{3x^2 - 2x - 8}$

(i) $\lim_{x \rightarrow \infty} \frac{\sqrt{17x^4 - 1}}{x^2 + x}$

(j) $\lim_{x \rightarrow -\infty} \frac{\sqrt{17x^4 - 1}}{x^2 + x}$

(k) $\lim_{x \rightarrow 0} x^3 \cos \frac{1}{x}$

4. Evaluate the following limit:

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$$

What is the function for which this limit will compute the slope of the tangent line at the point $x = 2$?

5. (a) Graph the function $2 \ln(t + 1) - 1$, compute the domain and range. Label carefully intermediate steps.
- (b) What are the vertical asymptotes?
- (c) Where is this function continuous?
- (d) What is the rate of change of $2 \ln(t + 1) - 1$ at the point $t = 0$. Use limits and Problem 6 from Homework assignment 3 to answer this question.

6. For the following functions, determine horizontal and vertical asymptotes if they exist. For each function state where the function is continuous.

(a) $f(x) = \frac{x^2+1}{x^2-1}$.

(b) $f(x) = \frac{x}{1+x}$.

(c) $f(x) = \sqrt{x+1} - \sqrt{x}$.

7. Let $P(t)$ denote the population of a community of squirrels at time t in years after the year 1999. Suppose that

$$P(t) = \frac{4000}{1 + Ca^t},$$

where C and a are positive constants and $0 < a < 1$. Suppose also that the population of squirrels in 1999 ($t = 0$) was 1000, and the population of squirrels now ($t = 5$) is 2000.

- (a) Determine the constants C and a .
- (b) When will the population of squirrels be 2500?
- (c) What is $\lim_{t \rightarrow \infty} P(t)$?
8. The price of a stock is modeled by a function $v(t) = Ae^{rt}$ of exponential type, where A and r are constants. Here, t is time in months after January 28, 2003. Assume the stock price is \$200 on January 28, 2003 and the price was \$50 on January 28, 2001 (i.e. exactly 24 months ago).
- (a) Find a formula for $v(t)$. Give EXACT answers for A and r .
- (b) What is the average rate of change in the stock price over the past 24 months? Give an EXACT answer and include units.
- (c) How long does it take the stock price to triple? (Round your answer to the nearest month.)
9. (a) Find the equation of the tangent line to the curve $y = x^3 - x$ at the point where $x = r$. For which values of r is this tangent line horizontal?
- (b) Find an equation of the tangent line to $y = e^x \sin x$ at the point $(0, 0)$.

10. The height $y(t)$ (in feet at time t seconds) of a ball thrown vertically upwards is given by $y(t) = -16t^2 + 128t + 25$. Find the velocity of the ball at time $t = 1$. Find the velocity of the ball when it hits the ground.