MIDTERM II
Math 124, Section C
May 16, 2006

- No book, notes or graphing calculators are allowed. You may use a scientific calculator.

- Show all your work to get full credit.

- Read instructions for each problem CAREFULLY.

- Check your work!
1. (12pts) Find the following derivatives. You do not have to simplify.

(a) \( f(x) = \tan\left(\frac{x^4}{\sqrt{17x^3+1}}\right) \), \( f(x)' = \)

(b) \( f(x) = x^{\cos x} \), \( f(x)' = \)

(c) \( y = \arccos(t) \), \( \frac{d^2y}{dt^2} = \)
2. (12pts) A (spherical) snowball is rolling down a snow covered hill in such a way that its radius is changing at the rate of 3 cm/min. Determine the rate of change of the volume of the snowball when the radius is 4 cm. Include units.

(You may use the formula for the volume of a sphere of radius $r$: $V = \frac{4}{3}\pi r^3$.)
3. (12pts) Consider the curve given by the equation

\[ y^2 = (x + 1)(x^2 - 1/2) \]

Use implicit differentiation to answer the following questions:

(a) Find all values of \( x \) such that the tangent line to the curve at the point \((x, y)\) is horizontal. How many such points are on the curve?

Note: you do not have to compute the values of \( y \).

(b) Find all points \((x, y)\) on the curve where the tangent line is vertical.
4. (a) Find an equation of the tangent line to graph of the function \( y = \sqrt[4]{x} \) at the point \((16, 2)\).

(b) Using linear approximation, estimate \( \sqrt[4]{17} \).

(c) Is your estimate below or above the actual value? Give a short graphical explanation.
5. A particle starts moving at the time \( t = 0 \). Its position at the time \( t \) is given by the parametric equations

\[
x(t) = \frac{2t}{t^2 + 1}, \quad y(t) = \frac{t^2 - 1}{t^2 + 1}
\]

(a) Find the coordinates of the position of the particle at the time \( t = 2 \).

(b) Compute horizontal and vertical velocities of the particle.

(c) Compute \( \frac{dy}{dx} \) as a function of \( t \).
(d) Find an equation of the tangent line to the trajectory at the time $t = 2$.

(e) Show that the tangent line at the point $(x(2), y(2))$ is perpendicular to the line connecting the origin with the point $(x(2), y(2))$.

6. (2pt) Bonus. Sketch and name the parametric curve in problem 5. Justify your answer. FULL CREDIT ONLY.