

MIDTERM II  
Math 124, Section C  
May 16, 2006

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	
6(Bonus)	2	

- No book, notes or graphing calculators are allowed. You may use a scientific calculator.
- Show all your work to get full credit.
- Read instructions for each problem CAREFULLY.
- Check your work!

1. (12pts) Find the following derivatives. You do not have to simplify.

(a)  $f(x) = \tan\left(\frac{x^4}{\sqrt[4]{17x^3+1}}\right)$ ,  $f(x)' =$

(b)  $f(x) = x^{\cos x}$ ,  $f(x)' =$

(c)  $y = \arccos(t)$ ,  $\frac{d^2y}{dt^2} =$

2. (12pts) A (spherical) snowball is rolling down a snow covered hill in such a way that its radius is changing at the rate of 3 cm/min. Determine the rate of change of the volume of the snowball when the radius is 4 cm. Include units.

(You may use the formula for the volume of a sphere of radius  $r$ :  $V = \frac{4}{3}\pi r^3$ .)

3. (12pts) Consider the curve given by the equation

$$y^2 = (x + 1)(x^2 - 1/2)$$

Use implicit differentiation to answer the following questions:

- (a) Find all values of  $x$  such that the tangent line to the curve at the point  $(x, y)$  is horizontal. How many such points are on the curve?

Note: you do not have to compute the values of  $y$ .

- (b) Find all points  $(x, y)$  on the curve where the tangent line is vertical.

4. (a) Find an equation of the tangent line to graph of the function  $y = \sqrt[4]{x}$  at the point  $(16, 2)$ .

(b) Using linear approximation, estimate  $\sqrt[4]{17}$ .

(c) Is your estimate below or above the actual value? Give a short graphical explanation.

5. A particle starts moving at the time  $t = 0$ . Its position at the time  $t$  is given by the parametric equations

$$x(t) = \frac{2t}{t^2 + 1}, \quad y(t) = \frac{t^2 - 1}{t^2 + 1}$$

- (a) Find the coordinates of the position of the particle at the time  $t = 2$ .

- (b) Compute horizontal and vertical velocities of the particle.

- (c) Compute  $\frac{dy}{dx}$  as a function of  $t$ .

(d) Find an equation of the tangent line to the trajectory at the time  $t = 2$ .

(e) Show that the tangent line at the point  $(x(2), y(2))$  is perpendicular to the line connecting the origin with the point  $(x(2), y(2))$ .

6. (2pt) Bonus. Sketch and name the parametric curve in problem 5. Justify your answer. FULL CREDIT ONLY.