Midterm I Answer Key

Math 124, C and D

April 21, 2006

1. (12pts) Evaluate the following limits:

   (a) $\lim\limits_{x\to 0} \frac{\sqrt{x+1}-2}{x}$

   Does not exist, $\lim\limits_{x\to +0} \frac{\sqrt{x+1}-2}{x} = -\infty$ and $\lim\limits_{x\to -0} \frac{\sqrt{x+1}-2}{x} = \infty$.

   (b) $\lim\limits_{x\to 1} \frac{x^2-2x+1}{x-1} = 1$

   (c) $\lim\limits_{x\to 0} \frac{\sin^2 x}{x} = 0$

2. (12pts) Consider the function $f(x) = \frac{\sqrt{2x^2+1}}{x-1}$

   (a) Find horizontal asymptotes of $f(x)$ or state that there are none.

   $\lim\limits_{x\to\infty} f(x) = \sqrt{2}$ and $\lim\limits_{x\to-\infty} f(x) = -\sqrt{2}$. Horizontal asymptotes at $\pm\sqrt{2}$.

   (b) Find vertical asymptotes of $f(x)$ or state that there are none.

   $x = 1$

   (c) Where is $f(x)$ a continuous function?

   $x \neq 1$

3. (12pts) Let $f(x) = x^2 - 2$

   (a) Find an equation of the tangent line to the graph of $f(x)$ at the point $x = 2$

   $m = \lim\limits_{h\to 0} \frac{f(2+h)-f(2)}{h} = 4$ is the slope, then using the point $(2, f(2)) = (2, 2)$ we find the equation to be

   $$y - 2 = 4(x - 2)$$

(b) Sketch the graph of $f(x)$. At which point would you expect the derivative of f to be 0? Sketch the tangent line to the graph at that point.

Tangent line will have its slope 0 at the vertex of the parabola.

4. (12pts) Here is the picture of the graph of a function $f(x)$.

(b) Find the following quantities for the new function $y(x)$:

(i) $y(10) = -19$

(ii) $y'(5) = -2$

(iii) $\lim\limits_{x\to 20} \frac{y(x)-y(20)}{x-20} = 2$

5. (12pts) The College of the Exciting on-campus living underwent a renovation project. On January 3rd of the year 2010 the college opened up a fantastic new cafeteria. The word quickly got out and the number of students eating at the new place each day was growing exponentially starting from the very first day. A number of professors got interested in the popularity of the new restaurant and conducted research which estimated that there are 5% more students coming each day compared to the previous day. The maximum capacity of the place is 1000. On the first day exactly quarter of the restaurant was filled. Estimate when the cafeteria will first exceed its capacity.

With $S(t) = 250e^{0.05t}$, set $S(t) = 1000$ and solve for t to find $t = \frac{\ln 4}{0.05} = 20\ln 4 \approx 27.73$ which means it would be January 30th. The other way to do this problem is to solve for the constant r by setting $S_0 = S_0 e^r \Rightarrow r = \ln 1.05$ which means that $t \approx 28.41$.