

MIDTERM I  
Math 124, Section D  
April 18, 2006

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	
6 (bonus)	5	

- No book, notes or graphing calculators are allowed. You may use a scientific calculator.
- Show all your work to get full credit.
- Read instructions for each problem CAREFULLY.
- There are 5 core problems, and 1 bonus problem. You should do bonus problem ONLY IF you complete the five core problems ahead of time.
- Check your work!

1. (12pts) Evaluate the following limits:

(a)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+2}-2}{x}$

(b)  $\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x-1}$

(c)  $\lim_{x \rightarrow 0} \frac{\sin^2(x)}{x}$

2. (12pts) Consider the function  $f(x) = \frac{\sqrt{2x^2+1}}{x-1}$

(a) Find horizontal asymptotes of  $f(x)$  or state that there are none.

(b) Find vertical asymptotes of  $f(x)$  or state that there are none.

(c) Where is  $f(x)$  a continuous function?

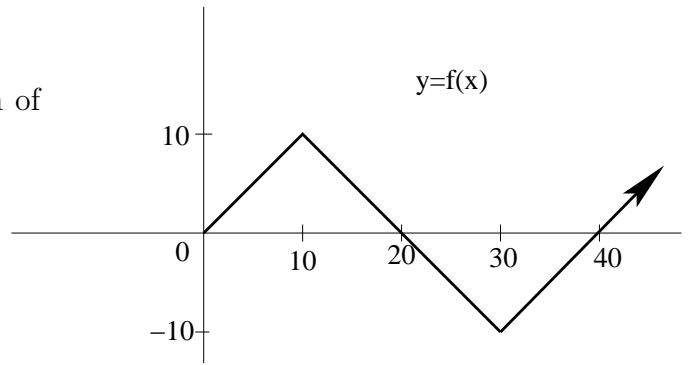
3. (12pts) Let  $f(x) = x^2 - 2$

(a) Find an equation of the tangent line to the graph of  $f(x)$  at the point  $x = 2$ .

Note: You must use limits when computing the slope: no credit for using shortcuts from chapter 3 or from your previous experience with calculus.

(b) Sketch the graph of  $f(x)$ . At which point would you expect the derivative of  $f$  to be 0? Sketch the tangent line to the graph at that point.

4.(12pts). Here is the picture of the graph of a function  $f(x)$ .



(a) Sketch the graph of the function  $y = -2f(x) + 1$ .

(b) Find the following quantities for the new function  $y(x)$ :

(i)  $y(10) =$

(ii)  $y'(5) =$

(iii)  $\lim_{x \rightarrow 20} \frac{y(x) - y(20)}{x - 20} =$

5. (12pts) The College of the “Exciting on-campus living” underwent a renovation project. On January 3<sup>rd</sup> of the year 2010 the college opened up a fantastic new cafeteria. The word quickly got out and the number of students eating at the new place each day was growing exponentially starting from the very first day. A number of professors got interested in the popularity of the new restaurant and conducted research which estimated that there are 5% more students coming each day compared to the previous day. The maximum capacity of the place is 1000. On the first day exactly quarter of the restaurant was filled. Estimate when the cafeteria will first exceed its capacity.

Note: As the problem suggests, you may assume that the number of students eating at the restaraunt grows according to the exponential model  $S(t) = Ce^{rt}$ . It will be convenient to measure the time  $t$  in days. You may leave your answer in the exact form.

6. (5pts) BONUS. Find the equation of the tangent line to the function  $y = \sqrt{4 - (x - 1)^2}$  at the point  $(2, \sqrt{3})$  *using precalculus only*.