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Quillen

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Modules of Constant Jordan type

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MSRI Evans Lecture, April 14, 2008

Atiyah-Swan conjecture

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Modules of Constant Jordan type D. Quillen, "The spectrum of an equivariant cohomology ring I, II," Ann. Math. 94 (1971)

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Modules of Constant Jordan type D. Quillen, "The spectrum of an equivariant cohomology ring I, II," Ann. Math. 94 (1971)

G - finite group, $k = \mathbb{F}_p^{\mathrm{alg}}$.

What is the Krull dimension of $H^*(G, k)$?

Atiyah-Swan conjecture

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G - finite group, $k = \mathbb{F}_p^{\mathrm{alg}}$.

What is the Krull dimension of $H^*(G, k)$?

Conjecture (Atiyah, Swan): Krull dim $H^*(G, k) = p$ - rank of G

Definition

$$p - \text{rank} = \max_{E \subset G} \text{rk } E$$

where $E \simeq \mathbb{Z}/p \times \mathbb{Z}/p \times \cdots \times \mathbb{Z}/p$ runs over all elementary abelian *p*-subgroups of *G*.

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Modules of Constant Jordan type Back up...

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Modules of Constant Jordan type

Theorem

The cohomology ring $H^*(G, k)$ is a graded commutative k-algebra.

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Modules of

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Jordan type

Theorem

The cohomology ring $H^*(G, k)$ is a graded commutative k-algebra.

$$\mathsf{H}^{\bullet}(G,k) = \begin{cases} \mathsf{H}^{*}(G,k), & \text{if } p = 2, \\ \mathsf{H}^{\text{ev}}(G,k) & \text{if } p > 2. \end{cases}$$

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Theorem (Venkov (1959), Evans (1961))

The cohomology ring $H^{\bullet}(G, k)$ of a finite group G is a finitely generated k-algebra.

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Modules of Constant Jordan type

Theorem (Maschke)

Let M be a representation of a finite group G over \mathbb{C} . Let $N \subset M$ be a G-invariant subspace. Then N splits off as a direct summand: $M = N \oplus N'$, N' is G-invarient subspace.

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Corollary. Every representation over $\mathbb C$ is *completely reducible* - a direct sum of simple modules.

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Corollary. Every representation over $\mathbb C$ is *completely reducible* - a direct sum of simple modules.

Modular representation theory: char $k \mid \#G$.

Representations are not completely reducible. Lots of non-split extensions (exact sequences of G-modules).

$$N \longrightarrow M \longrightarrow M/N$$

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Cohomology $H^*(G, k) \iff \text{extensions } [k \to \cdots \to k].$

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Modules of Constant Jordan type Cohomology $H^*(G, k) \iff \text{ extensions } [k \to \cdots \to k].$

• $H^{i}(G, k) = \operatorname{Ext}_{G}^{i}(k, k)$, additive group for every n.

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$$[k \to \cdots \to M_j \to k] \quad \circ \quad [k \to N_1 \to \cdots \to k]$$

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$$k \to \cdots \to M_i \to N_1 \to \cdots \to k$$

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Modules of Constant Jordan type Cohomology $H^*(G, k) \iff \text{ extensions } [k \to \cdots \to k].$

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•
$$H^*(G, k) = Ext_G^*(k, k) = \bigoplus_{i>0} Ext_G^i(k, k)$$

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$$k \to \cdots \to M_j \to N_1 \to \cdots \to k$$

•
$$H^*(G, k) = Ext_G^*(k, k) = \bigoplus_{i>0} Ext_G^i(k, k)$$

Remark. This gives the same cohomology ring as the one defined in Dave Benson's talk two weeks ago (in terms of projective resolutions and cup product).

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Modules of Constant Jordan type

•
$$E = (\mathbb{Z}/p)^{\times r}$$
, rk $E = r$

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Modules of Constant Jordan type

- $E = (\mathbb{Z}/p)^{\times r}$, rk E = r
- $H^*(E, k) = k[x_1, \ldots, x_r] \otimes \underbrace{\Lambda^*(y_1, \ldots, y_n)}$

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Modules of Constant Jordan type

Example [Cohomology of elementary abelian p-groups].

nilpotents

- $E = (\mathbb{Z}/p)^{\times r}$, rk E = r
- $\bullet \ \mathsf{H}^*(E,k) = k[x_1,\ldots,x_r] \otimes \underbrace{\Lambda^*(y_1,\ldots,y_n)}_{}$

• $H^*(E, k)_{red} = k[x_1, \dots, x_r]$

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Modules of Constant Jordan type

Example [Cohomology of elementary abelian *p*-groups].

- $E = (\mathbb{Z}/p)^{\times r}$, rk E = r
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- $H^*(E, k)_{red} = k[x_1, \dots, x_r]$
- Spec $k[x_1,\ldots,x_r]\simeq \mathbb{A}^r$

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Modules of Constant Jordan type

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- $H^*(E, k)_{red} = k[x_1, \dots, x_r]$
- Spec $k[x_1, ..., x_r] \simeq \mathbb{A}^r$ $\underbrace{(x_1 \lambda_1, ..., x_r \lambda_r)}_{\text{max ideal}} \leftrightarrow \underbrace{(\lambda_1, ... \lambda_r)}_{\text{point on } \mathbb{A}^r}.$

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Example [Cohomology of elementary abelian *p*-groups].

- $E = (\mathbb{Z}/p)^{\times r}$, rk E = r
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Definition (Support variety)

$$|G| = \operatorname{Spec} H^{\bullet}(G, k),$$

the **support variety** of G (set of prime ideals with Zariski topology).

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Example [Cohomology of elementary abelian p-groups].

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• $H^*(E, k)_{red} = k[x_1, \dots, x_r]$

• Spec $k[x_1, ..., x_r] \simeq \mathbb{A}^r$ $\underbrace{(x_1 - \lambda_1, ..., x_r - \lambda_r)}_{\text{max ideal}} \leftrightarrow \underbrace{(\lambda_1, ..., \lambda_r)}_{\text{point on } \mathbb{A}^r}.$

Definition (Support variety)

$$|G| = \operatorname{Spec} H^{\bullet}(G, k),$$

the **support variety** of G (set of prime ideals with Zariski topology).

Example. $|E| \simeq \mathbb{A}^r$, dim |E| = r.



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Modules of Constant Jordan type Roughly: |G| is "determined" by $|E| \subset |G|$, where $E \subset G$ runs over all elementary abelian p-subgroups of G.

 $E \subset G$

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Modules of Constant Jordan type

$$E \subset G$$
 \leadsto $H^{\bullet}(G, k) \to H^{\bullet}(E, k)$

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Modules of Constant Jordan type

$$E \subset G$$
 \hookrightarrow $H^{\bullet}(G, k) \to H^{\bullet}(E, k)$
 \hookrightarrow $res_{G,E} : |E| \to |G|$

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Modules of Constant Jordan type

$$E \subset G$$
 \leadsto $H^{\bullet}(G, k) \to H^{\bullet}(E, k)$
 \leadsto $\operatorname{res}_{G, E} : |E| \to |G|$ finite map

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Modules of Constant Jordan type

$$E \subset G$$
 $\hookrightarrow H^{\bullet}(G, k) \to H^{\bullet}(E, k)$
 $\hookrightarrow \operatorname{res}_{G, E} : |E| \to |G|$ finite map
 $\operatorname{res}_{G, E} |E| \simeq |E|/W_{E}$, where $W_{E} = N_{G}(E)/E$

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$$E \subset G$$
 \hookrightarrow $H^{\bullet}(G, k) \to H^{\bullet}(E, k)$
 \hookrightarrow $\operatorname{res}_{G, E} : |E| \to |G|$ finite map

$$\operatorname{res}_{G,E} |E| \simeq |E|/W_E$$
, where $W_E = N_G(E)/E$

Theorem (Quillen (weak form))

$$|G| = \bigcup_{E \subset G} \operatorname{res}_{G,E} |E|$$

Consequences

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Theorem (Quillen (weak form))

$$|G| = \bigcup_{E \subset G} \operatorname{res}_{G,E} |E|$$

Corollary (Atiyah-Swan conjecture)

$$\begin{aligned} \operatorname{Krull} \operatorname{\mathsf{dim}} \mathsf{H}^\bullet(G,k) &= \operatorname{\mathsf{dim}} \operatorname{\mathsf{Spec}} \mathsf{H}^\bullet(G,k) = \operatorname{\mathsf{dim}} |G| = \\ \operatorname{\mathsf{max}}_{E \subset G} \operatorname{\mathsf{dim}} |E| &= \operatorname{\mathsf{max}}_{E \subset G} \operatorname{\mathsf{rk}} E \end{aligned}$$

Consequences

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Corollary

Irreducible components of $|G| \leftrightarrow conjugacy$ classes of maximal elementary abelian subgroups

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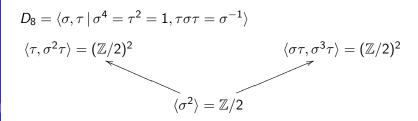
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Modules of Constant Jordan type $\textit{D}_8 = \langle \sigma, \tau \, | \, \sigma^4 = \tau^2 = 1, \tau \sigma \tau = \sigma^{-1} \rangle$

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Example: D_8

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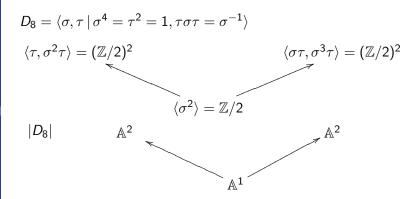
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Example: D_8

 $|D_8| \simeq \mathbb{A}^2 \times_{\mathbb{A}^1} \mathbb{A}^2$

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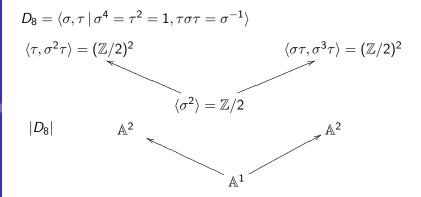
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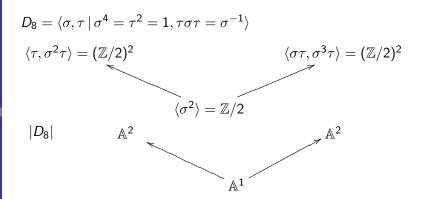
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$$|D_8| \simeq \mathbb{A}^2 \times_{\mathbb{A}^1} \mathbb{A}^2$$

Can check the answer because ...

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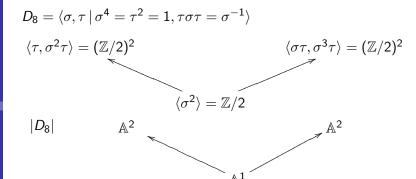
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Modules of Constant Jordan type



$$|D_8| \simeq \mathbb{A}^2 \times_{\mathbb{A}^1} \mathbb{A}^2$$

Can check the answer because ...

$$H^*(D_8, k) = k[x_1, x_2, z]/(x_1x_2)$$

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Modules of Constant Jordan type Alperin – Evens, Carlson, Avrunin – Scott.

A *G*-module $M \longrightarrow \text{a subvariety } |G|_M \subset |G|$.

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Modules of Constant Jordan type Alperin – Evens, Carlson, Avrunin – Scott.

A G-module $M \longrightarrow a$ subvariety $|G|_M \subset |G|$.

• $\operatorname{Ext}_G^*(M,M)$ is a ring (operations as for $\operatorname{Ext}_G^*(k,k)$).

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• $\mathcal{I}_M = \operatorname{Ker}\{H^{\bullet}(G,k) \to \operatorname{Ext}_G^*(M,M)\}$

Definition

The support variety of a G-module M

$$|G|_M = Z(\mathcal{I}_M) \subset |G|,$$

where
$$Z(\mathcal{I}_M) = \langle \wp | \mathcal{I}_M \subset \wp \rangle$$

Properties

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- $\bullet |G|_{M \oplus N} = |G|_M \cup |G|_N.$
- If $0 \to M_1 \to M_2 \to M_3 \to 0$ is a short exact sequence, then $|G|_{M_i} \subset |G|_{M_{i+1}} \cup |G|_{M_{i+2}}$.
- $\bullet |G|_{\Omega M} = |G|_{M}.$
- (Tensor product property) $|G|_{M \otimes N} = |G|_M \cap |G|_N$
- (Restriction) Let $H \subset G$, M a G-module. Then $\operatorname{res}_{G,H}(|H|_M) = |G|_M$.
- dim $|G|_M$ = complexity of M (= rate of growth of the minimal projective resolution).
- $|G|_{L_{\zeta}} = \langle \zeta \rangle$ a hypersurface in |G| defined by $\zeta = 0$, $\zeta \in H^{\bullet}(G, k)$.

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Modules of Constant Jordan type • Group algebra kGbasis as k-vector space: $\{e_g\}_{g \in G}$ multiplication: $e_g \cdot e_h = e_{gh}$

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• kG is a finite-dimensional Hopf algebra (has a coproduct: $kG \to kG \otimes kG$, $e_g \mapsto e_g \otimes e_g$)

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- Representations of $G \stackrel{\sim}{\longleftrightarrow} kG$ -modules
- Cohomology of $G \stackrel{\sim}{\longleftrightarrow}$ cohomology of kG

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Other algebraic structures that correspond to fin. dim-I Hopf algebras (and have theories of support vareities)

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Modules of Constant Jordan type Other algebraic structures that correspond to fin. dim-I Hopf algebras (and have theories of support vareities)

Lie algebras in char p

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- Finite group schemes (e.g., inifinitesimal subgroups of algebraic groups, such as GL_n)

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Theorem (Friedlander-Suslin, (1997))

Let A be a finite-dimensional co-commutative Hopf algebra over a field k of positive characteristic. Then the cohomology algebra $H^{\bullet}(A, k)$ is finitely generated.

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Modules of Constant Jordan type Let G be an algebraic group defined over k, g = Lie(G).

$$g \leftrightarrow u(g)$$
,

the restricted enveloping algebra of g.

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Assume p = char k is "big enough" (p > h).

Theorem (Friedlander-Parshall, Andersen-Jantzen (1983-84))

$$|g| = \mathcal{N}(g),$$

where $\mathcal{N}(g)$ is the nullcone of g, the variety of all nilpotent elements of g.

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Very different from finite groups! In particular, ${\cal N}$ is irreducible.

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Modules of

Constant

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Very different from finite groups! In particular, $\mathcal N$ is irreducible. Support varieties for modules \leftrightarrow theory of nilpotent orbits.

Representation theory of a finite group is usually "wild" - we

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cannot classify indecomposable modules.

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Modules of Constant Jordan type Representation theory of a finite group is usually "wild" - we cannot classify indecomposable modules.

Exception: $\mathbb{Z}/p = \langle \sigma \rangle$.

$$k\mathbb{Z}/p = \frac{k[\sigma]}{(\sigma^p - 1)} = \frac{k[\sigma]}{(\sigma - 1)^p} = \frac{k[t]}{t^p},$$

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Complete description of representation theory:

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k - simple module

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 \mathbb{Z}/p -module M

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 $\mathbb{Z}/p ext{-module }M \leftrightarrow \text{Jordan canonical form of }\sigma \text{ as an operator on }M$

Representation theory of the cyclic group \mathbb{Z}/p

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Representation theory of the cyclic group \mathbb{Z}/p

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Write additively:

$$k[t]/t^p$$
 - module $M \longleftrightarrow \mathsf{Jordan}$ type $a_1[1] + a_2[2] + \cdots + a_p[p]$

Representation theory of the cyclic group \mathbb{Z}/p

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Modules of Constant Jordan type

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Free
$$k[t]/t^p$$
-module $=\bigoplus_a k[t]/t^p \longleftrightarrow$ Jordan type $a[p]$.

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Modules of Constant Jordan type $E = (\mathbb{Z}/p)^{\times r}$. Choose generators g_1, \dots, g_r . Let $t_1 = g_1 - 1, \dots, t_r = g_r - 1$.

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$$E=(\mathbb{Z}/p)^{\times r}$$
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$$kE = k[g_1, \ldots, g_r]/(g_i^p - 1) = k[t_1, \ldots, t_r]/(t_i^p).$$

Definition

Let $\underline{\alpha} = (\alpha_1, \dots, \alpha_r) \in \mathbb{A}^r$. A shifted cyclic subgroup $<\underline{\alpha}>$ of E corresponding to $\underline{\alpha}$ is a cyclic subgroup of kE generated by a p-unipotent element $\alpha_1 t_1 + \dots + \alpha_r t_r + 1$.

Cyclic shifted sub-s are parametrized by the affine space \mathbb{A}^n_k .

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 V_E = variety of cyclic shifted subgroups.

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There is a natural isomorphism $V_E \simeq |E|$.

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Modules of Constant Jordan type

Definition (Carlson)

$$V_E(M) = \{\underline{\alpha} = (\alpha_1, \dots, \alpha_r) \in \mathbb{A}^r \mid \langle \alpha_1 t_1 + \dots + \alpha_r t_r + 1 \rangle$$
does not act freely on M }

- the **rank variety** of *M*.

Rank variety

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- the **rank variety** of *M*.

Theorem (Avrunin-Scott, (1982))

Let M be a finite-dimensional kE-module. The isomorphism $V_F \simeq |E|$ restricts to

$$V_E(M) \simeq |E|_M$$
.

Rank variety

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$$V_E(M) \simeq |E|_M$$
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Cyclic shifted subgroup is NOT a subgroup of E. It is a subgroup of kE.

π -points

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Modules of Constant Jordan type Cyclic shifted subgroup $\langle \underline{\alpha} \rangle = \langle \alpha_1 t_1 + \ldots + \alpha_r t_r + 1 \rangle$ of *E* determines a map of algebras

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Definition $(\pi\text{-point})$

A π -point α of a finite group G is a map of algebras

$$k[t]/t^p - - - \frac{\alpha}{2} - - > kC$$

which factors through some abelian p-subgroup $A \subset G$.

The map $kA \rightarrow kG$ is induced by a subgroup, the other two are just maps of algebras.

π -points

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Modules of Constant Jordan type • A π -point $k[t]/t^p \rightarrow kG \longrightarrow$

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$$H^{\bullet}(G, k) \rightarrow H^{\bullet}(k[t]/t^p, k) \simeq k[x] \sim$$

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• Spec $k[x] \rightarrow \operatorname{Spec} H^{\bullet}(G, k) = |G|$

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• $\mathbb{A}^1 = \operatorname{Spec} k[x] \to \operatorname{Spec} H^{\bullet}(G, k) = |G|$

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- $\mathbb{A}^1 = \operatorname{Spec} k[x] \to \operatorname{Spec} H^{\bullet}(G, k) = |G|$ Projectivize (factor out the scalar action of k^*):

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Some π - points \mapsto same point on Proj $H^{\bullet}(G, k)$

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Example.
$$E = \mathbb{Z}/p \times \mathbb{Z}/p$$
, $kE = k[t_1, t_2]/(t_1^p, t_2^p)$. $t \mapsto \alpha_1 t_1 + \alpha_2 t_2 \quad t \mapsto \alpha_1 t_1 + \alpha_2 t_2 + t_1^2$

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Equivalence relation on π -points

$$\alpha \sim \beta$$

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Equivalence relation on $\pi{\rm -points}$ $\alpha \sim \beta$

Solely in terms of local properties of representations

П-space

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Modules of Constant Jordan type

Definition (Π-space)

$$\Pi(G) = rac{\langle \pi - \mathsf{points} \quad lpha : k[t]/t^p
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angle}{\sim}$$

This is a topological space.

П-space

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Modules of Constant Jordan type

subgroups

Definition (Π-space)

$$\Pi(G) = \frac{\langle \pi - \mathsf{points} \quad \alpha : k[t]/t^p \to kG \rangle}{\sim}$$

This is a topological space.

Definition

Let M be a G-module.

$$\Pi(G)_M = <[\alpha]: k[t]/t^p \to kG: \alpha^*M \text{ is not free}>$$

 α^*M is a $k[t]/t^p$ – module where t acts via $\alpha(t) \in kG$.

M - finite dimensional $\mapsto \Pi(G)_M$ are precisely the closed sets of $\Pi(G)$.

theorem

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Modules of Constant Jordan type

Carlson's conjecture holds for Π -spaces:

Theorem (Friedlander-P.)

$$\Pi(G) \simeq \operatorname{\mathsf{Proj}} |G|$$

$$\underbrace{\Pi(G)_M}_{local\ prop} \simeq \underbrace{\operatorname{Proj}|G|_M}_{cohomology}$$

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M is projective $\Leftrightarrow \Pi(G)_M = \emptyset \Leftrightarrow M$ is free when restricted to any subalgebra $k[t]/t^p \to kG$.

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• Projectivity can be detected locally on π -points.

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Theorem (Detection of projectivity)

M is projective $\Leftrightarrow \Pi(G)_M = \emptyset \Leftrightarrow M$ is free when restricted to any subalgebra $k[t]/t^p \to kG$.

- Projectivity can be detected locally on π -points.
- Can replace kG by any finite dimensional co-commutative Hopf algebra

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• M is projective \Leftrightarrow at every π -point $\alpha: k[t]/t^p \to kG$ the Jordan type of M is a[p].

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- M is projective \Leftrightarrow at every π -point $\alpha: k[t]/t^p \to kG$ the Jordan type of M is a[p].
- ullet In particular, the Jordan type is the same at every $\pi ext{-point}.$

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- M is projective \Leftrightarrow at every π -point $\alpha: k[t]/t^p \to kG$ the Jordan type of M is a[p].
- In particular, the Jordan type is the *same* at every π -point.
- Are there other modules with this property?

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Modules of Constant Jordan type

- M is projective \Leftrightarrow at every π -point $\alpha: k[t]/t^p \to kG$ the Jordan type of M is a[p].
- In particular, the Jordan type is the *same* at every π -point.
- Are there other modules with this property?

Definition

M is a module of constant Jordan type if the Jordan type of M at every π -point $\alpha: k[t]/t^p \to kG$ is the same (the operator $\alpha(t)$ on M has the same Jordan canonical form for all α).

A very interesting and elusive class of modules!

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Modules of Constant Jordan type • If *M* is of constant Jordan type, then *M* is determined by this unique type.

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- If *M* is of constant Jordan type, then *M* is determined by this unique type.
- Which Jordan types can occur?

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- If *M* is of constant Jordan type, then *M* is determined by this unique type.
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- Some constructible examples:

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- Quite limited!

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Conjecture. [Carlson-Friedlander-P.] Let p>3, dim |G|>1. The type [2]+n[p] does not occur.

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Conjecture. [Carlson-Friedlander-P.] Let p > 3, dim |G| > 1. The type [2] + n[p] does not occur.

Theorem (D. Benson, March 2008)

Let G be a finite group, dim |G| > 1. There does not exist a G-module of constant Jordan type $[\mathbf{a}] + \mathbf{n}[\mathbf{p}]$ for $2 \le a \le p - 2$.