What is computation?

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therisingsea.org

1407.2650 “Logic and linear algebra”

1406.5749 “On Sweedler’s cofree cocommutative coalgebra”

1402.4541 “Computing with cut systems”

- Turing machines, Lambda calculus, logic…
- Semantics : syntax :: representations : group
- Homotopy type theory (Awodey, Voevodsky 2012)
- Girard “Towards a Geometry of Interaction” (1989)

Sense & Denotation

- Frege "On sense and denotation" (1892)
- A sentence *denotes* or refers to some external object, and expresses its *sense*, which is the 'mode of presentation' of its denotation.

$$2 \times 2 = 4$$

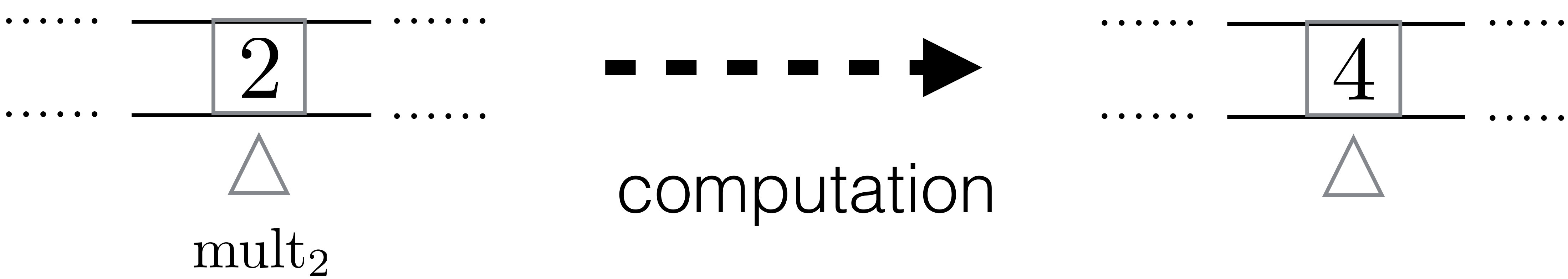
same denotation, different sense

Sense as algorithm

$$2 \times 2 = 4$$

$$\mathrm{mult}_2(2) = 4$$

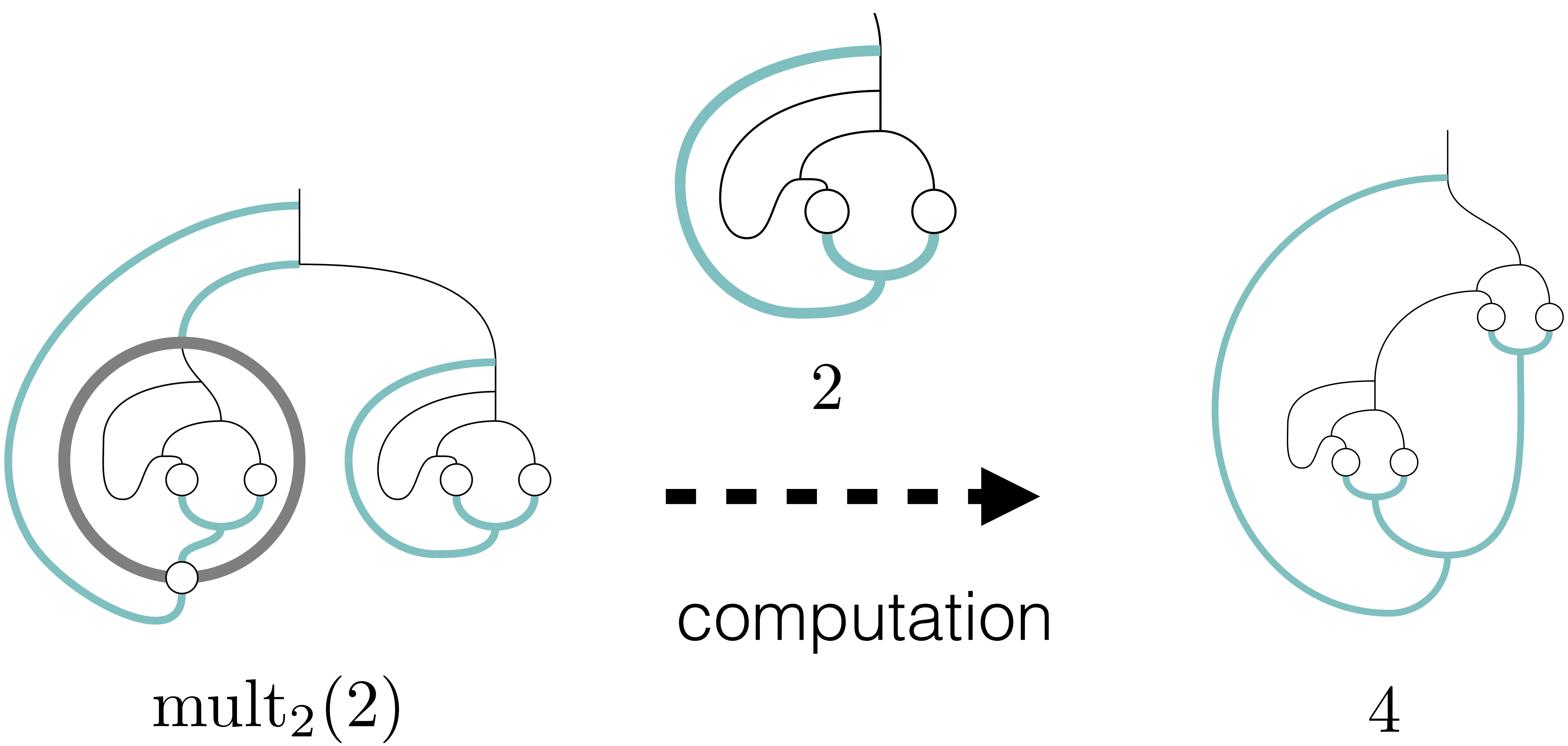
Turing machine input output



Sense as topology

$$2 \times 2 = 4$$

proof-nets = diagrammatics of linear logic



Sense as algebra

$\mathcal{T} = \mathbb{Z}_2$-graded triangulated category $\quad [1] \circ [1] = \mathrm{id}$

$$\mathrm{End}^*_{\mathcal{T}}(Y) = \mathrm{Hom}_{\mathcal{T}}(Y, Y) \oplus \mathrm{Hom}_{\mathcal{T}}(Y, Y[1])$$

$C = \mathbb{Z}_2$-graded algebra

A C-module in $\mathcal{T}$ is a morphism $C \longrightarrow \mathrm{End}^*_{\mathcal{T}}(Y)$

$\mathcal{T} = \mathbb{Z}_2$-graded triangulated category $\quad [1] \circ [1] = \mathrm{id}$

Example

$$C_1 = \mathrm{End}_k(k \oplus k[1]) \qquad a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad a^\dagger = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$= k\langle a, a^\dagger \rangle \text{ with } a^2 = (a^\dagger)^2 = 0, aa^\dagger + a^\dagger a = 1$$

$$C_n = k\langle a_1, \ldots, a_n, a_1^\dagger, \ldots, a_n^\dagger \rangle \text{ with Clifford relations}$$

$\mathcal{T}^\bullet = C_n$-modules in $\mathcal{T}$ for $n \geq 0$

$C_0 = k$

Sense as algebra

$\mathcal{T} = \mathbb{Z}_2$-graded triangulated category

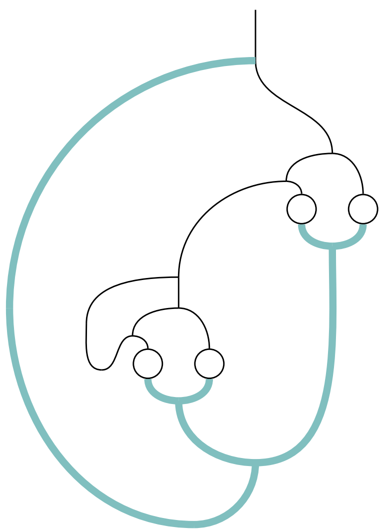
A C_1-module in $\mathcal{T}$ is $(Y, a, a^\dagger)$

$Y \cong X \oplus X[1] \quad X = \mathrm{Im}(aa^\dagger)$

$(Y, a, a^\dagger) \cong X \qquad 2 \times 2 = 4$

same denotation, different sense

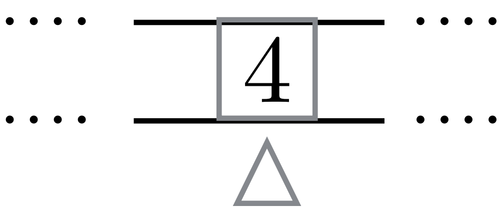
- A bicategory has objects, 1-morphisms and 2-morphisms, and composition functors



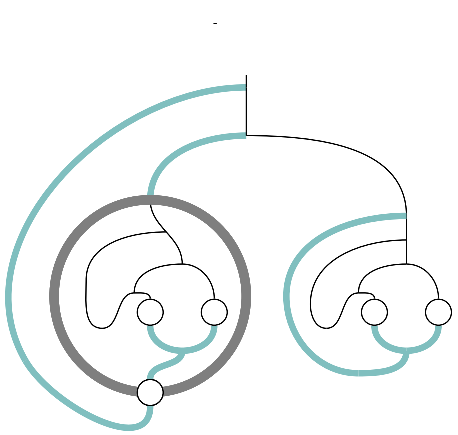
4

$$\mathcal{B}(b,c) \times \mathcal{B}(a,b) \longrightarrow \mathcal{B}(a,c)$$

$$(\mathbf{mult}_2, 2) \mapsto 4 = X$$



- A cut system is similar, except it has *cut functors*

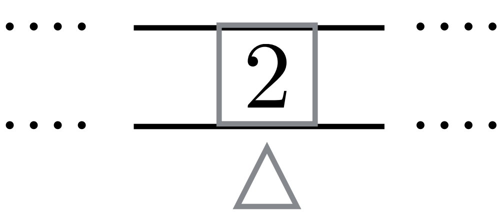


$\mathrm{mult}_2(2)$

$$\mathcal{B}(b,c) \times \mathcal{B}(a,b) \longrightarrow \mathcal{B}(a,c)^\bullet$$

(computable)

$$(\mathbf{mult}_2, 2) \mapsto (Y, a, a^\dagger)$$



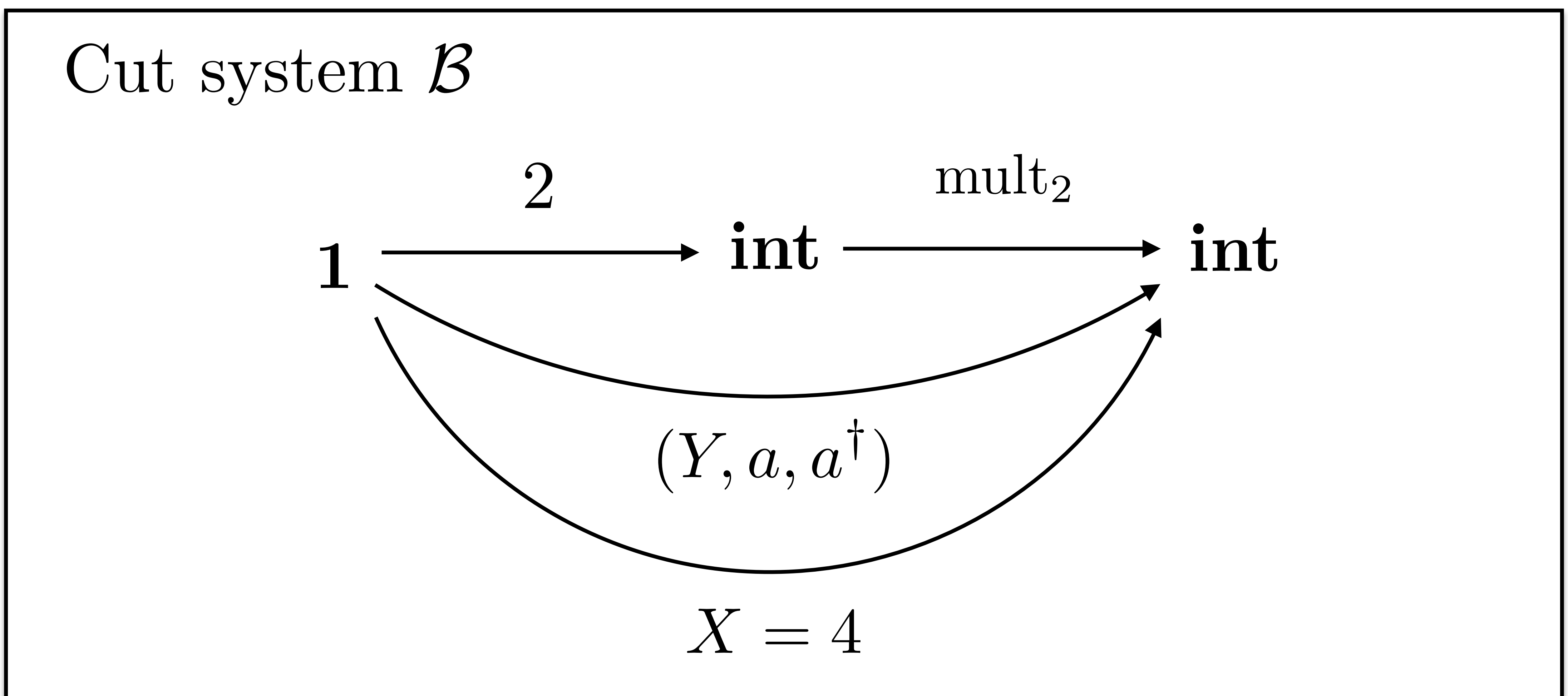
mult_2

Theorem

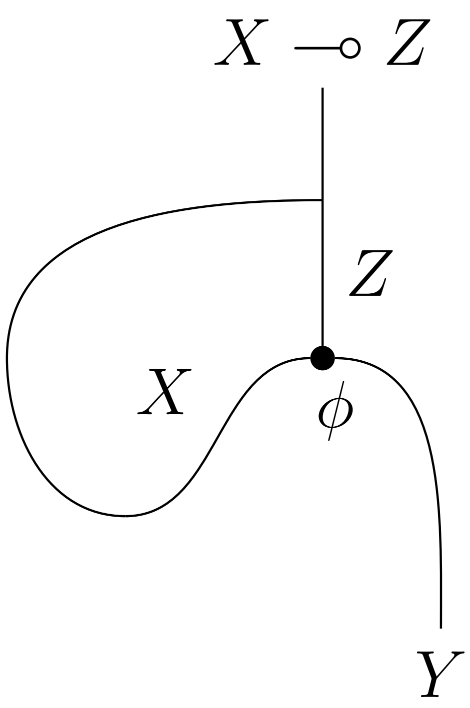
- There is a bicategorical semantics of intuitionistic propositional linear logic in the cocompletion of a cut system $\mathcal{B}$ defined on the bicategory of Landau-Ginzburg models (hypersurface singularities and matrix factorisations).
- Lambda calculus embeds in intuitionistic linear logic
- The Clifford actions are derived from Atiyah classes of matrix factorisations (homological perturbation lemma under the hood).

- Universal examples of same denotation, different sense: Turing machines, proof-nets, Clifford representations in triangulated categories (?)

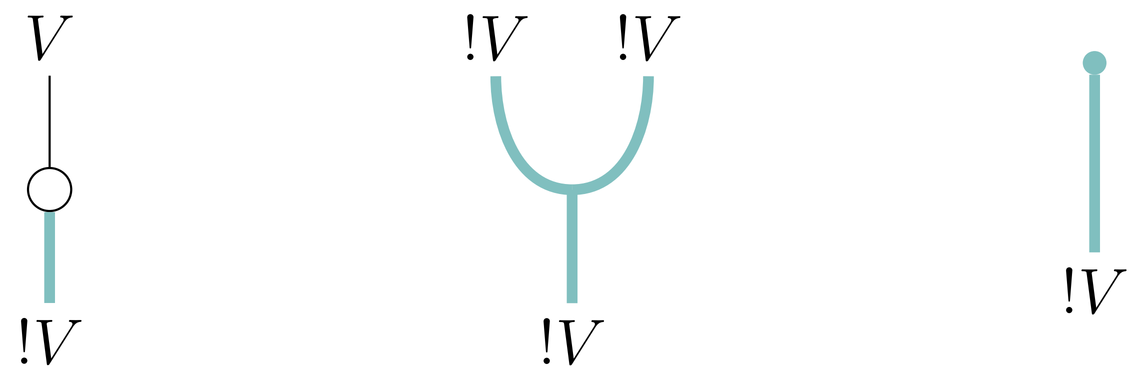
$$(Y, a, a^\dagger) \cong X \qquad 2 \times 2 = 4$$



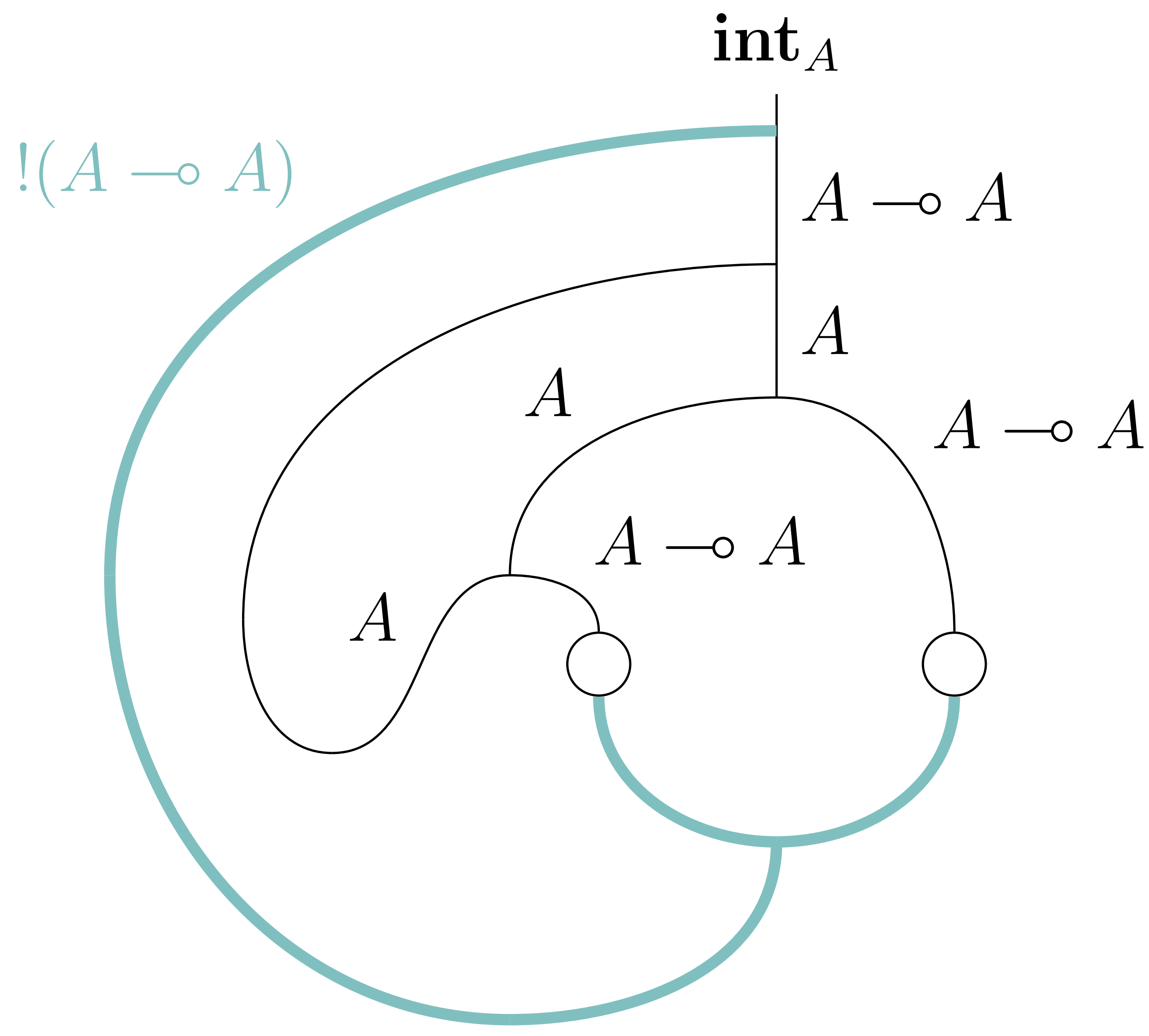
The adjoint $Y \longrightarrow X \multimap Z$ of a morphism $\phi : X \otimes Y \longrightarrow Z$ is depicted



$!V$ = universal coalgebra over V



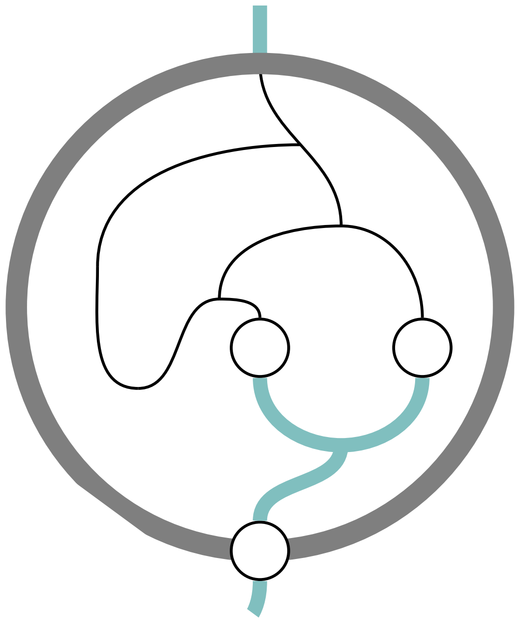
$$\mathbf{int}_A = !(A \multimap A) \multimap (A \multimap A) \qquad \alpha \mapsto \alpha^2$$

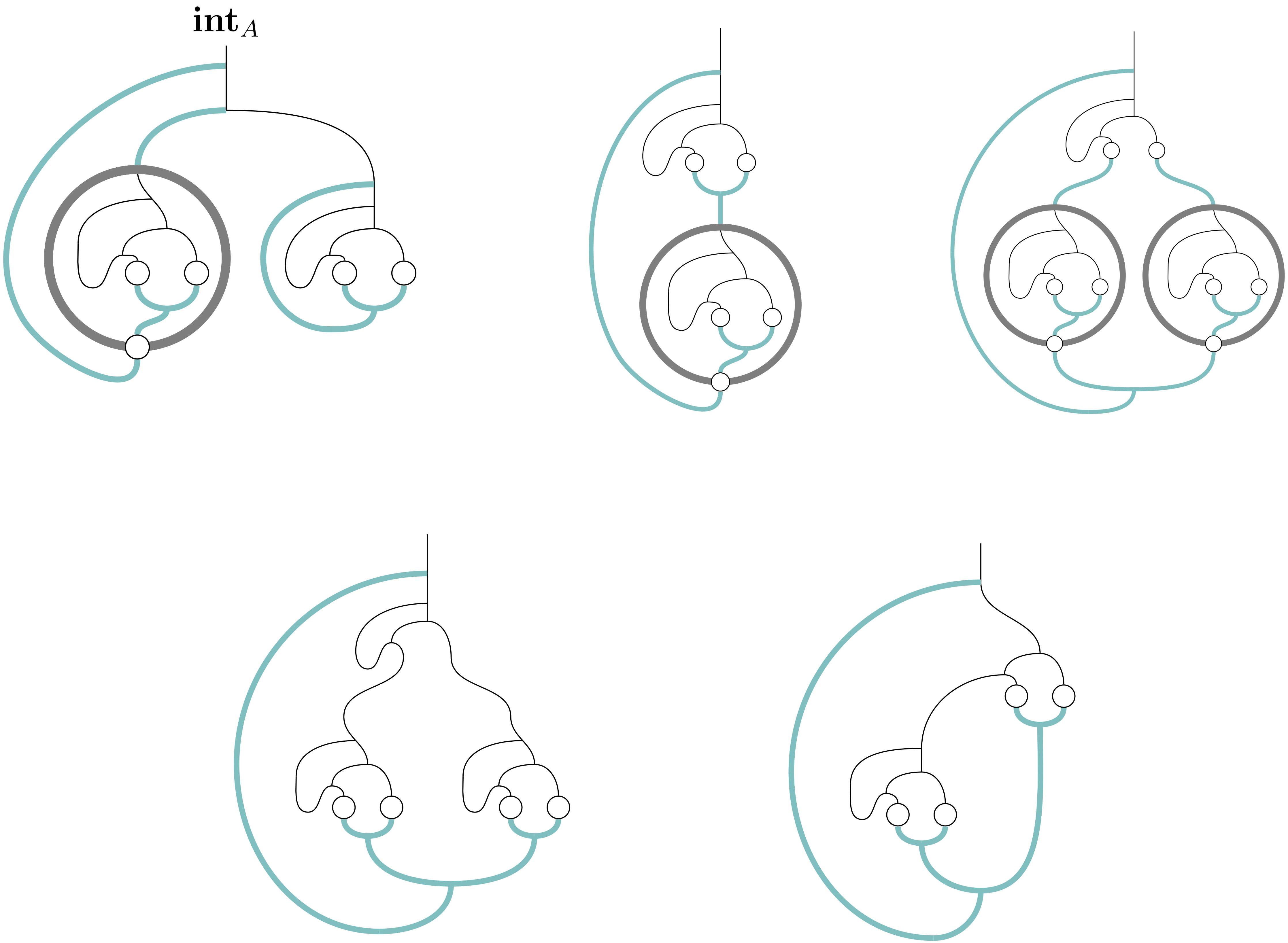


2

$$2 : !(A \multimap A) \longrightarrow (A \multimap A)$$

$$\widetilde{2} : !(A \multimap A) \longrightarrow !(A \multimap A)$$




$\mathbf{int}_A$