Research Statement

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Background

My research is in **derived algebra**, **homotopy theory** and **category theory**. This can be described as generalized abstract algebra and algebraic geometry, like Galois theory, invariant theory and topos theory, after sets have been repleaced by either topological spaces or chain complexes. This vein of investigation is also sometimes called "brave new algebra." Some of the most powerful invariants in algebra and topology, like homotopy groups, motivic cohomology, algebraic K-theory, and even Khovanov knot homology, have all been shown to be intrinsically "homotopical" objects, and as a result, derived algebra has been immensely useful in studying them (cf. [Voe11; Qui10; DGM13; LLS17]).

By understanding such invariants, we have been able to resolve long standing problems like the Bloch-Kato conjecture, the cobordism hypothesis and the Kervaire invariant one problem. Moreover, these methods have indicated new relationships between heretofore disconnected ideas in topology, number theory, algebraic geometry, mathematical physics and differential geometry. These developments would be almost impossible to achieve without the machinery of derived algebra in one manifestation or another (e.g. Quillen model categories or ∞-categories). Especially important to note is that classical algebraic structures like commutativity and associativity must be replaced (if one has any hope of recovering interesting examples) by structures which are parameterized by **operads**, which are of central importance to my research.

Often, invariants of interest, like those mentioned above, cannot be computed directly, but must be understood by piecing together "local data," akin to studying the geometry of a manifold or scheme by way of an atlas and transition functions (though the manifold in this case is often replaced by an entire category). This process of gluing is often referred to as *descent theory*. My research is concerned with understanding two aspects of this process. First, how can we take objects which are *not* naturally spaces, like differential graded algebras or monoidal categories, and think of them as spaces (so that we may understand them globally by piecing together suitably "local" data); second, in the case that we do understand an object locally, how can we *descend* that data to global information. Two places that this is made explicit are in my study of the Galois theory of bordism rings ([Bea17b; Bea18]) and in my ongoing work aimed at describing a version of the Zariski spectrum for Poisson algebras and algebras equipped with derivations (described in the final section of this statement).

A natural to place pursue the above program is the homotopy theorist's *category of spectra* $\mathcal{S}$. This category contains all discrete abelian groups, rings, and their modules, but additionally contains all stable homotopy types, chain complexes, and DGAs. On top of this, every so-called *extraordinary cohomology theory* can be identified with a spectrum and can now interact in new ways with classical objects. In other words, the category of spectra allows mathematicians to compare algebraic and topological objects on the same footing. Of particular importance is a certain ring spectrum called the *sphere spectrum*, $\mathbb{S}$, which has the property that every other spectrum is a module over it. There is even a ring spectrum map $\mathbb{S} \to \mathbb{Z}$ providing a "lower" base ring over which all discrete algebra occurs (cf. my paper with Morava [BM18]).

Major Projects

An area of particular interest to me is studying the descent properties of ring maps of the form $R \to R/\!\!/G$, where $R/\!\!/G$ is a suitably derived (or stacky) quotient of a ring object R by a G-action, where G can be a group or something more general like a loop space. A major thrust of my research has been to understand such maps as **Hopf-Galois extensions**, which are a model for principal bundles in noncommutative geometry. By working in the derived setting, certain classical ring maps, like the quotient map $\mathbb{Z} \to \mathbb{F}_2$, are Hopf-Galois extensions, though this structure is invisible in classical algebra. To be precise, in this setting we can say that $Spec(\mathbb{F}_2)$ is a $Spec(B\mathbb{Z})$-bundle over $Spec(\mathbb{Z})$, where $B\mathbb{Z}$ is a *topological* bialgebra (i.e. an $\mathbb{E}_\infty$-bialgebra in simplicial sets), rather than a discrete one.

An H-Hopf-Galois extension $A \to B$, for a bialgebra H, can be understood as describing $Spec(B)$ as a principal $Spec(H)$-bundle over $Spec(A)$, even in the case that A, B and H are not commutative [Sch04; Sch90]. Naturally, given the fundamental nature of principal bundles, this idea has played a central role in noncommutative geometry (and thus in associated disciplines like quantum algebra and representation theory [Mon09; Rum98; Sch99]). Hopf-Galois extensions also have a computationally useful descent theory. Given an H-Hopf-Galois extension $A \to B$, one can readily construct descent spectral sequences to compute invariants of A (e.g. Hochschild (co)homology and algebraic K-theory) in terms of those invariants' values on B and H (c.f. [DR17; Rot09; Şte95]). One of the goals of my research is to generalize this structure when working with derived generalizations of rings, e.g. algebras over operads, ring spectra, $\mathbb{A}_\infty$-categories or monoidal ∞-categories. I have had success with this goal in certain cases, as described in [Bea18; BM18; Bea17b].

In a slightly different direction, I also study the **Balmer spectrum**, an invariant of triangulated categories which has found applications in representation theory, algebraic topology and algebraic geometry (cf. [Bal05; Bar+17]). This invariant, when applied to the derived category of a commutative ring, recovers the Zariski spectrum of that ring. In ongoing work with J. Zhang, I am studying the the result of applying the Balmer spectrum functor to an algebra over a more general operad. Of particular interest is computing the Balmer spectra of commutative rings with derivation, e.g. $(k[x], \frac{d}{dx})$, and using them as the "spaces" of noncommutative geometry. We are also interested in understanding the Balmer spectra of algebras over the shifted Poisson operads, which are relevant to questions arising in quantization and mathematical physics (cf. [MS18; Saf18]). In working with operadic structure, it appears to be useful to apply homotopy theoretic methods, and one major goal of this work is to construct a general but computable approach to noncommutative and Poisson geometry.

Finally, much of the above work requires developing new technology in higher category theory and proving that it behaves as one would expect. My collaborator L.Z. Wong and I have written two papers that describe and simplify the so-called **Grothendieck construction** in enriched category theory, which we are in the process of using to better understand noncommutative derived algebra ([BW18a; BW18b]). I also have several long term research goals in pure category theory that include developing the theory of ∞-operads in the ∞-cosmoi of Riehl and Verity [RV]; giving a categorical construction of Koszul duality in terms of generalized Reedy categories [BM11] that includes the Grothendieck construction, Thom spectra, and classical and operadic Koszul duality; and constructing higher categorical generalizations of ∞-operads which allow for "operations" that admit a space of inputs (rather than a finite set).

Past Work

Thom Spectra Are Hopf-Galois Extensions

Heretofore my work has mostly been concerned with doing derived algebra in the setting of $\mathbb{E}_k$-ring spectra for $k < \infty$, where $\mathbb{E}_k$ is the *little k-cubes operad.* The results of [Bea17b; Bea18] produce many new examples of Hopf-Galois extensions formed by taking quotients.

Theorem 1 ([Bea17b; Bea18]). *Let G be a k-fold loop space acting on an $\mathbb{E}_{k+1}$-monoidal connective ring spectrum R. Then $R/\!\!/G$ is an $\mathbb{E}_k$ comodule over the $\mathbb{E}_k$-coalgebra $R[BG] = R \otimes BG_+$ and the quotient map $R \to R/\!\!/G$ is an $R[BG]$-Hopf-Galois extension of $\mathbb{E}_k$-R-algebras, up to a completion of R.*

The above theorem, unlike [Rog08] and [Rot09], takes into account *both* multiplicative and comultiplicative structure up to coherent homotopy, and generally produces a larger family of examples. In particular, every $\mathbb{E}_k$-monoidal bordism spectrum admits this structure. The above also yields a pair of fascinating examples with strong connections to algebra: there are Hopf-Galois extensions $\mathbb{S} \to H\mathbb{Z}$ and $\hat{\mathbb{S}}_2 \to H\mathbb{F}_2$ (cf. [ACB14; BM18]). The above theorem can also be used to give an ∞-categorical, and operadic, framework for understanding coTHH and A-theory of spaces as in [Boh+17; HS14].

Half a Galois Correspondence for Quotients of Ring Spectra

Hopf-Galois extensions generalize Galois extensions of rings [KT81; Mon09]. It is natural then to ask, given an H-Hopf-Galois extension $A \to B$, does there exist a *Galois correspondence*? In other words, given a sub-algebra $J \subseteq H$, is there an intermediate sub-extension $A \to C \to B$ with J as the Hopf-algebra for the first map and H/J as a the Hopf-algebra for the second? In general, finding conditions under which such a correspondence exists is quite hard, even for discrete rings [Mon09]. However, the results of [Bea17b; Bea18] give one direction of such a correspondence in the general case of a quotient ring spectrum $R/\!\!/G$ and a "subgroup" $H \subset G$. Notice that here, as above, G need not be a topological group, but only an iterated loop space.

Theorem 2 ([Bea17b; Bea18]). *Let G be a $k+1$-fold loop space acting on an $\mathbb{E}_{k+1}$ monoidal ring spectrum R and $F \to G \to K$ be a fibration of $k+1$-fold loop spaces. Then there is a composite Hopf-Galois extension of $\mathbb{E}_{k-1}$ ring spectra: $R \to R/\!\!/F \to R/\!\!/G$. The Hopf-algebra of $R/\!\!/F \to R/\!\!/G$ is the $\mathbb{E}_{k-1}$-monoidal cocommutative R-bialgebra $R[BK]$.*

In other words, the fibration $F \to G \to K$ induces a fibration $BF \to BG \to BK$ and this is a manifestation of the fact that BF is "normal" enough for $BG/BF \simeq BK$ to still be a Hopf-algebra. The Galois correspondence here then would be the map from sub-algebras of G (equivalently BG) to intermediate extensions of $R \to R/\!\!/G$. In topology, working with bordism spectra, the identification of such objects results in new Künneth-type theorems like $MSpin \wedge_{Mstring} MSpin \simeq MSpin \wedge \mathbb{S}[K(\mathbb{Z},4)]$ (cf. [Bea17b] for many more examples). In the context of noncommutative geometry the above equivalence says that $MSpin$ is a $Spec(K[\mathbb{Z},4])$-torsor over $MString$.

Applying this theorem to the final example of the previous section gives an interesting flavor to the composition $\mathbb{S} \to H\hat{\mathbb{Z}}_2 \to H\mathbb{F}_2$. Recall that both $H\hat{\mathbb{Z}}_2$ and $H\mathbb{F}_2$ are extensions of $\mathbb{S}$ with Hopf-algebras $\Omega^2 S^3\langle 3\rangle$ and $\Omega^2 S^3$ respectively. As a 3-connective cover, $\Omega^2 S^3\langle 3\rangle$ is the fiber of a map $\Omega^2 S^3 \to S^1$. Thus by the theorem, we have a Hopf-Galois extension $H\hat{\mathbb{Z}}_2 \to H\mathbb{F}_2$ for which the Hopf-algebra is $S^1 \simeq B\mathbb{Z}$. Note that this phenomenon is not visible unless one allows for Hopf-algebras which are topological in nature, like $B\mathbb{Z}$ (again cf. [BM18]). One of the goals of my research is to study this phenomenon more generally for localizations and completions of noncommutative rings and their derived generalizations.

Chromatic Homotopy Theory

The above work was originally inspired by chromatic homotopy theory, and despite finding applications elsewhere, has continued to be useful in that setting. A major theorem of chromatic homotopy theory is the so-called Nilpotence Theorem of Devinatz, Hopkins and Smith [DHS88]. The proof of the Nilpotence Theorem relies on understanding a sequence of (somewhat mysterious) $\mathbb{E}_2$-ring spectra $\{X(n)\}_{n\geq 1}$ whose colimit is the well known complex cobordism theory MU. The results of [Bea17a; Bea17b] give a new construction of the spectra $X(n)$ by attaching an $\mathbb{A}_\infty$-cell to $X(n-1)$. A first step in doing so is understanding the effects of attaching structured cells to ring spectra along an element in homotopy. If that element is $\alpha \in \pi_d(A)$, then the associated object with an $\mathbb{E}_k$-structured cell attached along α is denoted $A/\!\!/_{\mathbb{E}_k}\alpha$. The following theorem from [Bea17a] allows one to control, in a range, the homotopy groups of this quotient:

Theorem 3 ([Bea17a]). *Let A be a connective $\mathbb{E}_{k+1}$-ring spectrum and choose an element $\alpha \in \pi_d(A)$ for $d \geq 0$ with associated self-map $\bar{\alpha}\colon \Sigma^d A \to A$. Then there is a map of A-modules $cof(\bar{\alpha}) \to A/\!\!/_{\mathbb{E}_k}\alpha$ which induces an isomorphism on homotopy groups in degrees less than $2d$, where $A/\!\!/_{\mathbb{E}_k}\alpha$ denotes the spectrum A with an $\mathbb{E}_k$-cell attached along α.*

I use the above theorem to gain a better understanding of the homotopy groups of $X(n)$ and then prove that, after localizing at a prime, $X(n)$ can be produced from $X(n-1)$ in a canonical way:

Theorem 4 ([Bea17a]). *The spectrum $X(n)_{(p)}$ can be obtained from $X(n-1)_{(p)}$ by attaching an $\mathbb{A}_\infty$-$X(n)_{(p)}$-cell along any generator of the cyclic group $\pi_{2n-1}(X(n)_{(p)})$.*

I have also been able to compute the topological Hochschild homology of $X(n)$, as well at its cotangent complex with respect to $X(n-1)$ [Bea17b; Bea15]:

Theorem 5 ([Bea15; Bea17b]). *The topological Hochschild homology of $X(n)$ over $\mathbb{S}$ is equivalent to $X(n) \wedge \Sigma^\infty_+ SU(n)$, and the cotangent complex of $X(n)$ as an $X(n-1)$-algebra is equivalent to $\Sigma^{2n-2} F_{\mathbb{E}_1}(X(n-1)) \wedge_{X(n-1)} X(n)$.*

The above represents a beginning at attempting to understand these spectra from the perspective of derived algebraic geometry and deformation theory. In particular, combined with the results of [Bea17a; Bea17b], one can prove that $X(n)$ is a $Spec(\mathbb{S}[x])$-torsor over $X(n-1)$. There is much more to be done in understanding these spectra, considering that they have been shown in [DHS88; Rav86; Rav02] to be closely related to formal algebraic geometry and the stable homotopy groups of spheres. I have two different ongoing projects, one with Salch, and one with Lawson and Hahn, to better understand these spectra and their localizations. In the former, we are able to precisely identify the p-local obstructions to giving a complex orientation as certain permanent cycles of a spectral sequence described in [Rav02]. In the latter, we prove that Ravenel's spectra $T(n)$, which are split summands of the p-localizations of $X(n)$, admit an $\mathbb{A}_\infty$-structure.

Category Theory

When proving the above theorems, I often happened upon purely category theoretic results that were of independent interest. Specifically, with coauthor L.Z. Wong, I have given a detailed description of an enriched Grothendieck construction, inverse Grothendieck construction and a correspondence between enriched fibrations and pseudofunctors [BW18a]. This result fills a significant lacuna in the category theoretic literature. The Grothendieck construction, described in detail in, e.g. [Joh02], is useful in relating stacks and fibered categories, but no complete generalization to the enriched setting has appeared in the literature until now. Our main theorem is:

Theorem 6 ([BW18a]). *Let V be a monoidal category satisfying the assumptions of [BW18a, §4], and let B_V be the free V-category on a category B. There is a 2-equivalence*

$$I\colon OpFib(B_V) \cong Fun^{ps}(B, VCat) \colon Gr$$

between the 2-category of V-enriched opfibrations over B_V and the category of pseudofunctors from B to V-enriched categories.

By restricting to the case that $V = sSet$, so $VCat = sCat$, in [BW18b], we obtain a more elementary description of Lurie's *relative nerve* construction in the case of fibrations over a *discrete* category:

Theorem 7 ([BW18b])**.** *Let $F\colon D \to sCat$ be a functor, and $f = NF$, the simplicial nerve of F. Then there is an isomorphism of coCartesian fibrations over $N(D)$.*

$$N(GrF) \cong N_f(D).$$

where N_f is the relative nerve of D decribed in [Lur09].

Future Work

Derived Hopf-Galois Extensions from Quotients

My primary research objective is to generalize the cobordism or Thom spectra (i.e. quotient spectra) constructions of [ABG15; And+14] to derived noncommutative geometry and algebra. These constructions have proven incredibly useful in algebraic topology and homotopy theory, and can be directly transported to other objects like DGAs, $\mathbb{A}_\infty$-algebras, DG-categories, and tensor categories. The category theory is effectively formal, and already constructed in the above references. I would prove theorems like the following:

Conjecture 1. *Let $\mathcal{C}$ be an $\mathbb{E}_k$-monoidal ∞-category and let A be an $\mathbb{E}_k$-algebra in $\mathcal{C}$. If G is a k-fold loop space acting on A via A-module automorphisms then there is an $\mathbb{E}_{k-1}$-monoidal Hopf-Galois extension $A \to A/\!\!/G$ with Hopf-algebra $A[BG]$.*

One example of particular interest is the case of the action of a group G on an $\mathbb{A}_\infty$-algebra A as in [Zba09]. Then the induced quotient map $A \to A/\!\!/G$ can be thought of as a principal $Spec(A[BG])$-bundle of formal graded noncommutative manifolds as in [KS09]. However, the ∞-category in the above theorem could be: chain complexes over a k-algebra for a field k; dg-modules over a DGA; the ∞-category of monoidal ∞-categories; simplicial sheaves on any site, etc.

Still following [Zba09], we can further think of homotopy group actions on $\mathbb{A}_\infty$-algebras as being associated to local systems of $\mathbb{A}_\infty$-algebras on manifolds. Such local systems are often associated to solutions of the Maurer-Cartan equation for a certain dg-Lie algebra on M.

Conjecture 2. *Let M be a manifold with a local system of $\mathbb{A}_\infty$-algebras on it associated to a solution of a Maurer-Cartan equation as in [Zba09]. Then there is a functor $\mathcal{F}$ from the smooth ∞-groupoid $\mathcal{M}$ of M to the ∞-category of chain complexes such that for $x \in M$, $\mathcal{F}(x)$ is equivalent to the value of the local system at x. Moreover, there is a homotopy coherent action of the smooth loop object $\Omega\mathcal{M}$ on $\mathcal{F}(x)$ for all $x \in M$, and a derived $B\Omega\mathcal{M}$-Hopf-Galois extension $\mathcal{F}(x) \to \mathcal{F}(x)/\!\!/\Omega\mathcal{M}$.*

This avenue of research leads to a simple to understand categorical framework for computing twists of DeRham and Hochschild homology and cohomology. Indeed, this is a specialization of the fact that Thom spectra can be thought of as computing twisted generalized homology and cohomology, e.g. in [ABG10]. Additionally, Conjecture 2 can be generalized to replace manifolds with noncommutative spaces (where this can be interpreted in a number of different ways). In that case, one considers local systems of $\mathbb{A}_\infty$-algebras on ∞-categories of modules over noncommutative rings and DGAs.

Applications to Hochschild (co)homology, cyclic homology and algebraic K-theory

The above considerations allow one to apply technology from noncommutative geometry to many more examples that were previously unavailable. For instance, given a discrete Hopf-Galois extension $A \to B$ in rings with Hopf-algebra H, there is a spectral sequence that allows one to compute the Hochschild cohomology of A (which governs deformations of A), from the Hochschild cohomology of H with coefficients in the Hochschild cohomology of B. Since in the derived case H will typically be a space, we must replace Hochschild cohomology with *topological* Hochschild cohomology:

Conjecture 3. *Given a Hopf-Galois extension $A \to B$ in a symmetric monoidal ∞-category $\mathcal{C}$ with Hopf-algebra H, there is a spectral sequence of signature:*

$$E_2^{s,t} = THH^s(H, THH^t(B)) \Rightarrow THH^{s+t}(A)$$

.

In this way new computational tools will be introduced for computing Hochschild cohomology of DGAs, $\mathbb{A}_\infty$-algebras and discrete rings that arise as derived quotients. This should be compared to [FP98; Şte95], though again, my result is significantly more general and also holds for "higher" versions of Hochschild cohomology, in the sense of [Fra13]. Following [DR17], if $A \to B$ is a Hopf-Galois extension which is an isomorphism on π_0 and a surjection on π_1 then one can also expect good behavior for topological Hochschild *homology*, topological cyclic homology, and algebraic K-theory:

Conjecture 4. *Let $A \to B$ be an H-Hopf-Galois extension in a suitable ∞-category $\mathcal{C}$ (e.g. one which is symmetric monoidal, complete and admits a t-structure). Assume A and B are connective and the map $A \to B$ is 1-connected. Let F be one of the functors THH, TC or K. Define a cosimplicial algebra of $\mathcal{C}$ by setting $Y^n = B \otimes H^{\otimes n}$. Then there is a Bousfield-Kan spectral sequence of signature:*

$$E_1^{s,t} = \pi^s \pi_t F(Y^\bullet) \Rightarrow \pi_{t-s} F(A).$$

Hence the determination of new derived Hopf-Galois extension allows the application of homotopical methods to computations of important invariants of rings.

An Operadic Approach to Noncommutative and Poisson Geometry

The so-called *Balmer spectrum* $Sp(C)$ of a tensor triangulated category C, as described in [Bal12; Bal16; Bal05], is an often computable invariant that has recently found applications in equivariant homotopy theory, representation theory, noncommutative algebra and algebraic geometry. In the case that the tensor triangulated category C is the derived category of a commutative ring, $Sp(C)$ recovers the Zariski spectrum of R. More generally however, it is possible to produce the "derived category" of an $\mathcal{O}$-algebra A, $D_{\mathcal{O}}(A)$, where $\mathcal{O}$ is an operad (cf. [BM09]). In the case that $\mathcal{O}$ is a so-called *Hopf operad* and admits an operad morphism from commutative operad, this derived category will be tensor triangulated. So a natural goal in this setting is to compute the Balmer spectra of such derived categories $Sp(D_{\mathcal{O}}(A))$. It is not hard to prove that such objects exist:

Theorem 8. *Let k be a field of characteristic zero, $\mathcal{O}$ be an operad under Comm, and A an $\mathcal{O}$-algebra in $Vect_k$. Then there is a functorial construction $A \mapsto Sp(D_{\mathcal{O}}(A))$ which, in the case that $\mathcal{O} = Comm$, recovers the Zariski spectrum of A.*

Of special interest in this project, which is joint with Zhang, are the operads *CommDer*, defined in [Lod10], whose algebras are commutative k-algebras equipped with a derivation, and P_n, the n-shifted Poisson operad (cf. e.g. [Saf18]). We expect that the Balmer spectrum in these cases will be a subspace of the Zariski spectrum of the underlying commutative algebra (or its set of connected components) but equipped with additional structure. In the case of $\mathcal{O} = CommDer$, this would take the form of a vector field on the Balmer spectrum of the underlying ring. This can be generalized to allow algebras with more than one derivation. In the case of $\mathcal{O} = P_n$ we expect this extra structure to be a generalized symplectic foliation.

In general, the problem of computing Balmer spectra for operadic algebras leads to a number of new questions that we are excited to address. Poisson geometry is closely connected to mathematical physics and topological field theories, so it is of great interest to see to what degree the Balmer spectrum of a Poisson algebra can recover the associated Poisson manifold, Lie algebroid or shifted symplectic stack. We also aim to give geometric interpretations of the prime thick tensor ideals (the "points") of the Balmer spectrum in terms of localizations. Indeed, each such tensor ideal corresponds to a localization of the category of *operadic* modules over an $\mathcal{O}$-algebra A, and it is unclear what relationship such localizations have to the localizations of the category of modules over the underlying commutative algebra of A.

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