Mathematics and AI (Fall 2025): Jarod Alper
Lecture 4

What is mathematics good for? Does doing mathematics lose value when computers get better than us?

Is the referee system broken? Can we reimagine refereeing in the age of AI and formalization?

How might Tao's pre-rigorous / rigorous / post-rigorous stages change with formalization?

Should we still teach long division? Should we teach integration rules? Is there an enfeeblement risk of incorporating AI in mathematics?

A technical argument by a trusted author, which is hard to check and looks similar to arguments known to be correct, is hardly ever checked in detail.

—Vladimir Voevodsky

What is mathematics? What are mathematicians?

Ian Stewart

But I am drifting off the topic. What is mathematics? In despair, some have proposed the definition "Mathematics is what mathematicians do." And what are mathematicians? "People who do mathematics." This argument is almost Platonic in its perfect circularity. But let me ask a similar question. What is a businessman? Someone who does business? Not quite. It is someone who *sees opportunities* for doing business when others might miss them.

A mathematician is someone who sees opportunities for doing mathematics.

I'm pretty sure that's right, and it pins down an important difference between mathematicians and everyone else. What is mathematics? It is the shared social construct created by people who are aware of certain opportunities, and we call those people mathematicians. The logic is still slightly circular, but mathematicians can always recognize a fellow spirit. Find out what that fellow spirit does; it will be one more aspect of our shared social construct.

Welcome to the club.

What are mathematics helpful for?
Mathematics are helpful for physics. Physics helps us make fridges. Fridges are made to contain spiny lobsters, and spiny lobsters help mathematicians who eat them and have hence better abilities to do mathematics, which are helpful for physics, which helps us make fridges which...

—Laurent Schwartz

The main application of Pure Mathematics is to make you happy.

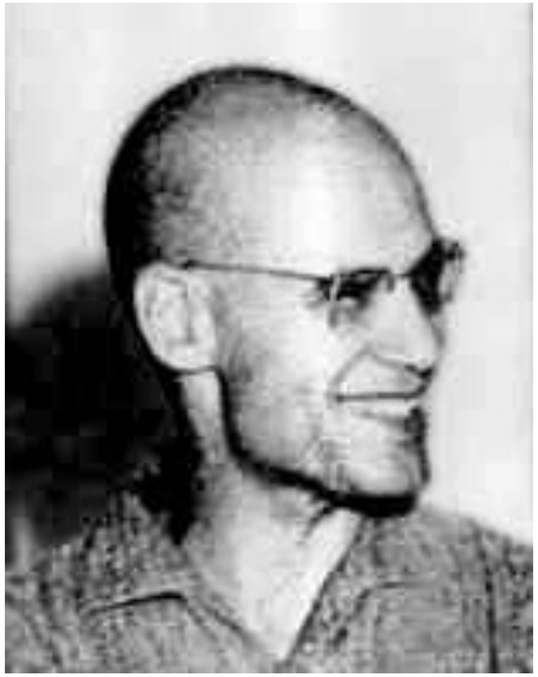
—H. Lenstra

What is AI?

“There are two quite different paradigms for AI. Put simply, the logic-inspired paradigm views sequential reasoning as the essence of intelligence and aims to implement reasoning in computers using hand-designed rules of inference that operate on hand-designed symbolic expressions that formalize knowledge. The brain-inspired paradigm views learning representations from data as the essence of intelligence and aims to implement learning by hand-designing or evolving rules for modifying the connection strengths in simulated networks of artificial neurons.” — Bengio, LeCun, Hinton

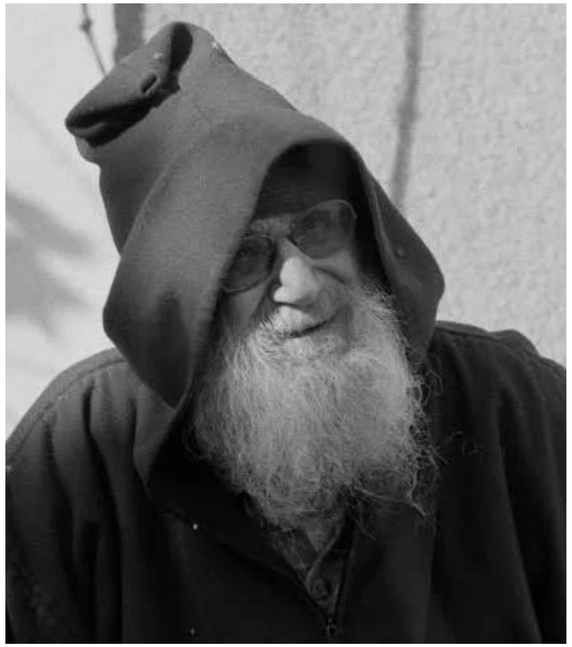
Terence Tao’s three distinct phases

Such an event would lead us to an
era during which the practice of our art will
be reduced to arid and vain demonstrations of
cerebral 'weightlifting', and to intellectual bidding wars
towards the 'breaking open' of competition problems.
—Alexander Grothendieck



To fear the error and to fear the truth are one and the same. One who fears to be wrong is powerless to discover.

—Alexander Grothendieck



Discussion Topics

Group 1: What is mathematics good for? What value do we receive from mathematics? Are we at risk of losing these values?

Group 2: Is the referee system broken? Can we reimagine refereeing in the age of AI and formalization?

Group 3: How might Tao's pre-rigorous / rigorous / post-rigorous stages change with formalization?

Group 4: Should we still teach long division? Should we teach integration rules? Is there an enfeeblement risk of incorporating AI in mathematics?