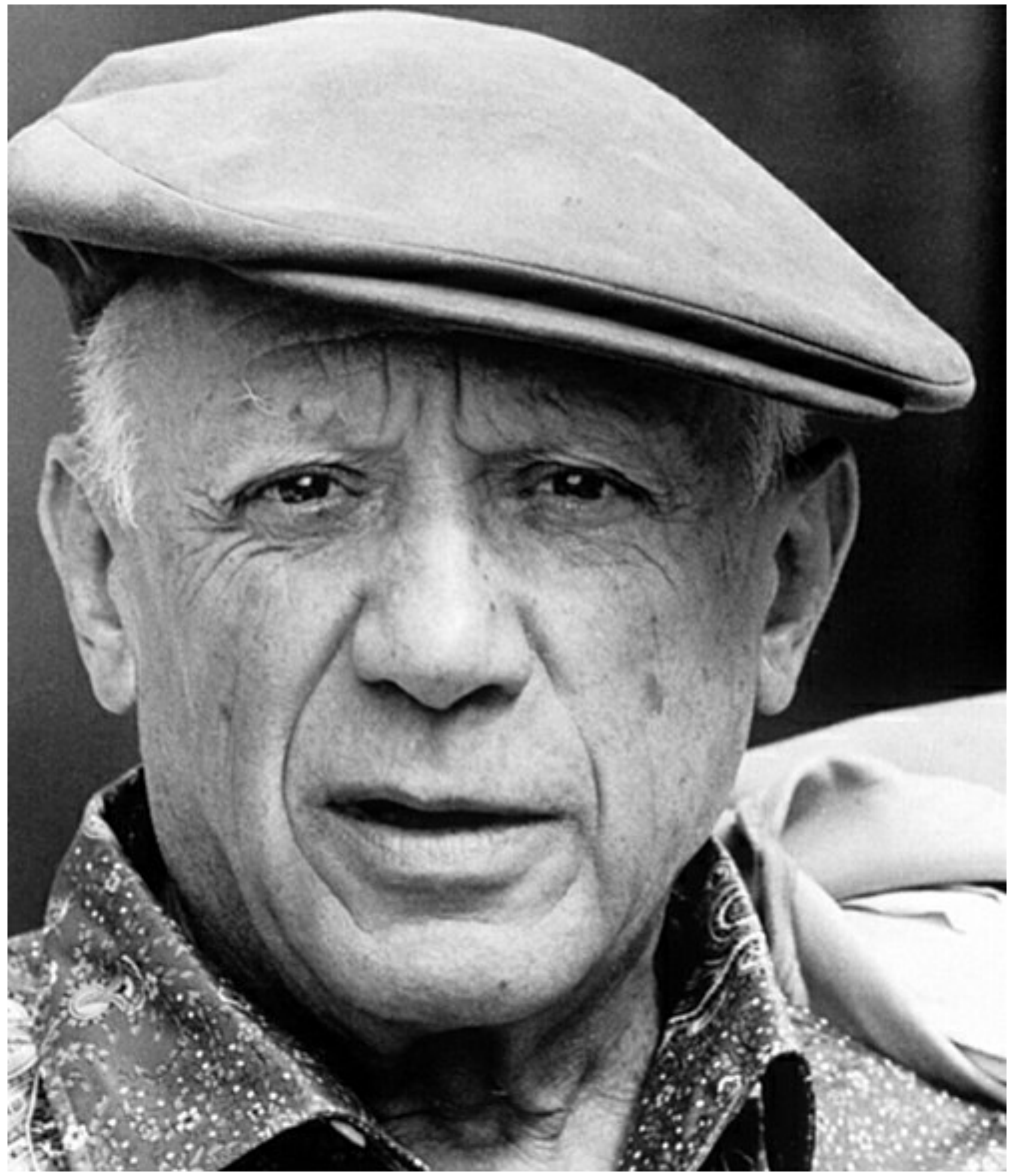
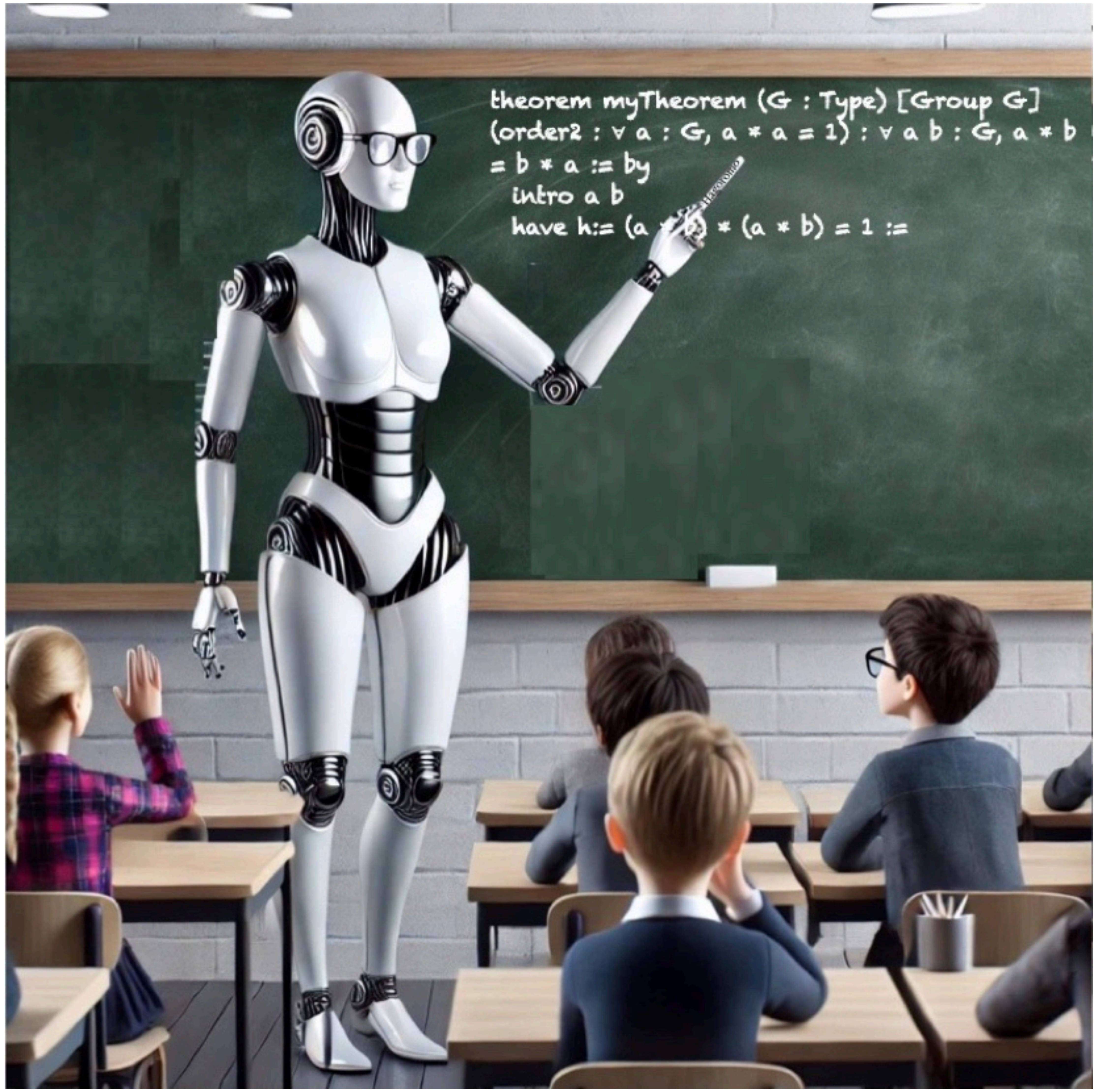
Mathematics and AI (Fall 2025): Jarod Alper
Lecture 1



Computers are useless. They can only give you answers.
—Pablo Picasso (1968)

What is Math & AI?

1. Mathematics behind AI
2. Formalization
3. Autoformalization
4. Machine learning to assist research
5. Mathematics in the age of AI
6. AI to accelerate human learning



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Mathematics & AI

Investing in applied machine learning without understanding the mathematical foundations is like investing in health care without understanding biology.
—Rebecca Willett (2023)

Formalization

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- *Mathematical formalization* is the process of writing a mathematical proof into a formal language that can be checked by a computer.
- This is achieved using a *proof assistant*, an interactive program that facilitates the translation of a proof into a sequence of logical deductions from the axioms.
- There are many proof assistants: Lean , Agda, Coq, Mizar, HOL, Isabelle, ...
- This is typically a very labor intensive process, although more sophisticated *tactics* are making it easier.
- Lean's mathematical library mathlib contains ~80% of undergraduate math
- Major success: Liquid Tensor Experiment

Why formalize?

> A word of warning - and apology. There are several thousand formulas in this paper which allow one or more `sign-like ambiguities'... I have made a superhuman effort to achieve consistency and even to make correct statements: but I still cannot guarantee the result. — David Mumford (1966)

Reasons

- ↳ Complete confidence in correctness
- ↳ Improving understanding
- ↳ Training computers
- ↳ Mathematical exposition
- ↳ Software verification

Autoformalization


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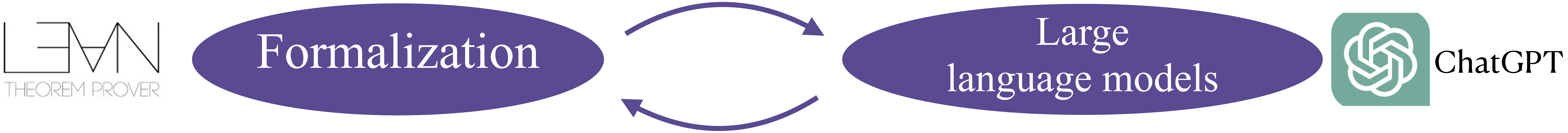
- *Autoformalization* is the formalization of natural language mathematics with minimal human input.
- *Automated theorem proving* is formalizing with no human input.
- The formal:informal proof ratio
  - Current ratio is between 100 – ∞, depending on field.
  - Goal is to get the ratio less than 1.
- Recent success:
  - Numina's solution to the 2024 AIMO progress prize
  - Deepmind's Silver-level 2024 IMO performance (using formal reasoning in AlphaProof)
  - Open AI, Deepmind (informal reasoning), Harmonic, ByteDance, and SeedProver have all announced impressive results on the 2025 IMO.

Combined power of formalization and AI

↳ Machine learning (ML) algorithms and large language models (LLMs) are statistical models that cannot always be trusted.



More reliable: train computers to give an answer *together* with a formalized proof.

↳ Provides a check against the hallucination of LLMs.

↳ Great potential for reinforced learning algorithms.

General consensus in mathematical community: in order for the community to accept an AI's solution to a longstanding conjecture, it better be formalized.

AlphaZero AlphaProof

- *AlphaZero,* developed by Google's DeepMind, uses probabilistic tree search and a deep neural network, which is trained to not only to learn a good valuation function of a board state but also a probability distribution called a *policy* for effective next moves.

- By replacing moves with logical steps and board states with proof states, these same techniques apply to proof generation.

- *AlphaProof* is Google DeepMind's autoformalization approach

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State of the art: Prime Number Theorem (PNT)

Prime Number Theorem (Dirichlet): The number of primes $\pi(x)$ less than x satisfies:
$$\pi(x) \sim \frac{x}{\log x}.$$

↳ 2006: Formalized in Isabelle by Avigad, Donnelly, Gray, and Raff

↳ July 2024: Formalized ‘MediumPNT’ in Lean by Tao, Kontorovich, and dozens of others

↳ September 2024: A collaboration between Morph Labs (Jeremy Han) and Mathinc’s Gauss (Christian Szegedy) autoformalized ‘StrongPNT’ using a very detailed blueprint

— 5k LOC Latex → 20k LOC Lean

Machine learning in mathematical research

- In the 1770s, Felkel and Vega computed factorization tables up to 408,000. This inspired Legendre and Gauss to conjecture the *prime number theorem*.
- In the 1960s, Birch and Swinnerton-Dyer used a primitive computer to count solutions to elliptic curves over finite fields, leading to their famous conjecture.

Machine learning algorithms of today are a massively superpowered version of this.

What have they been good at?

- Using large data sets to find relationships, e.g., *elliptic curve murmurations* (He, et al) and knot invariants (Davies, et al)
- Generate counterexamples for conjectures, e.g., graph theory (Wagner)
- Produce efficient formulas/representations, e.g., tensor decompositions for matrix multiplication (AlphaTensor)

Meaning of mathematics in the age of AI

- What do we do when computers outperform humans in the Olympiad?
- What about when they become better at proving theorems?
- What if they discover a five page elementary proof of Fermat's Last Theorem?
- What if they become better at generating conjectures or synthesizing mathematics?
- What if they become better at teaching mathematics? What if they become better at writing mathematics? What if they become better at understanding mathematics? What if they become better at being mathematicians? What if they become better at being human? What if they become better at being? What if they become? What if they? What if? What?

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