4/3/2019

Turing Machines

Def:

Let $\{0,1\}^*$ = set of all things in binary of finite length.

abbreviated as lang.

Def: A language $\subset \{0,1\}^*$ (ie lang is subset of $\{0,1\}^*$)

Ex: (of elements of $\{0,1\}^*$)

$\{0,1\}^* = \{\sqcup, 0, 1, 00, 01, \ldots\}$

Can be modified so that lang. $\subset \Sigma^*$

Ex: (of lang.)

Def. even = $\{n \geq 0, n \in \mathbb{Z}$, written in bin $|$ n is even$\}$

prime = $\{p \geq 2$, p is prime integer written in binary beginning w/ $1\}$

Note:

More generally, can consider any set Σ (called an alphabet) & we set Σ^* = set of all strings in Σ of finite length.

Ex:

$\Sigma = \{0, 1, 2\}$ $0212 \in \Sigma^*$

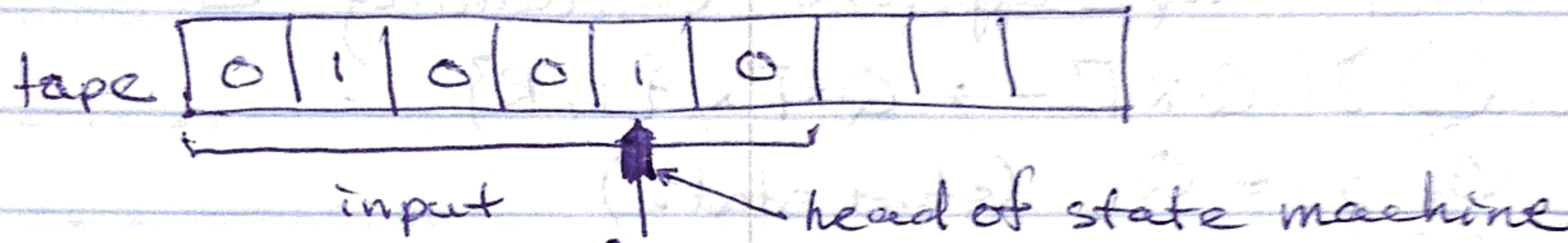
$\Sigma = \mathbb{Z}$

$\Sigma = \{a, b, \ldots, z\}$

GOAL: Given lang. $A \subset \Sigma^*$, we would like comp. model for determining if a given string $w \in \cancel{\emptyset} \Sigma^*$ is in A.

Conceptual def:

A Turing machine is



finite state machine
has a finite set called
states.

& finite set of rules which
govern action dep. on
current state & what head
reads from tape.

Action that can be taken: ~~later~~

a) print symbol from Σ

b) head moves to left/right

c) move to a different state

d) terminate w/ "accept" or "reject"

abbreviated as Txtbk

Precise def: (Def. 3.1 in Sipser Textbook)

A Turing machine = 7-tuple

$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$

1. Q is a finite set (states)
2. Σ = input alphabet
3. Γ = tape alphabet containing "$\sqcup$"
4. $\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$
5. $q_0 \in Q$ (start state)
6. ~~$q_a \in Q$ (~~ $q_{accept} \in Q$
7. ~~$q_a \in Q$~~ $q_{reject} \in Q$

Functions as follows:

Sps it's in state $q \in Q$ & sps it reads a $\gamma \in \Gamma$

$$\delta(q, \gamma) = (q', \gamma', L \text{ or } R)$$

Key defn:

Let $A \subset \Sigma^*$

Turing Machine M recognizes A if for any $w = w_1 w_2 \cdots w_n \in \Sigma^*$, then M accepts $w \iff w \in A$

Defn:

lang. is reg. if $\exists$ turing machine M, which recog. lang.