MATH 300 - Introduction to Mathematical Reasoning, Spring 2022 - Homework 1 Solution

Name: Jonathan Andrés Niño Cortés

Section 1.1 Q4

Solution:

For Part a, here is an assortment of correct answers I found while grading.

- GROAN – GROWN – GROWS – GLOWS – CLOWS – CLOTS – CLOTH
- GROAN – GROWN – CROWN – CLOWN – CLOWS – CLOTS – CLOTH
- GROAN – GROAT – GLOAT – BLOAT – BLEAT – BLEST – BLAST – BLASH – SLASH – SLOSH – SLOTH – CLOTH

For part b, one solution that works for the word "XYLEM" is to observe that there is no valid English word that can be constructed from the word XYLEM by changing only one letter. Therefore, there cannot be a chain of deductions starting from GROAN and ending in XYLEM.

A more general solution involves calculating the set of all true propositions (those that can be reach from GROAN) and check that XYLEM (or any other word) is not in this set. Many people made references to constructing a graph where the words are the nodes and there is an edge between two words if they differ in exactly one word. Then you can implement algorithms that explore the graph to find out if there is a path from GROAN to XYLEM.

I found an implementation of this in this link if you want to play with it. Try the words SLUSH and JUICE to see what happens.

Funny remark: FALSE is a true proposition.

Section 1.3 Q8

Solution:

- P: Claudia has trained properly.
- Q: Claudia is injury-free at race time.
- R: Claudia will run the marathon.

(a) $(P \vee Q) \Rightarrow R$.

(b) $R \Rightarrow Q$.

(c) $R \Leftrightarrow (P \wedge Q)$.

Many people made the mistake of using "$\Leftarrow$ " or "$\Leftrightarrow$" in part b. Note that Claudia will run the marathon only if Claudia is injury-free at race time is false only when: 1) she runs the marathon and 2) she is not injury free.

Section 1.3 Q10

Solution:
In general you want to add parenthesis when there is ambiguity on what operation is being calculated first in an expression.
For example many people added the parenthesis

$$((\sim P) \vee Q).$$

This is fine but leaving them out is also fine since $\sim$ has precedence over all the other logical operators. This means that it is understood that you apply negation first and then the OR.

a) Statement form

b) Not in statement form: $(((P \Rightarrow Q) \Rightarrow R) \Rightarrow S) \Rightarrow T$

c) Statement form

d) Not in statement form: $((P \Rightarrow (Q \Rightarrow R)) \wedge Q) \vee (\sim P \vee Q)$.
NOTE: If you missed this I invite you to calculate the truth tables of $((P \Rightarrow Q) \Rightarrow R)$ and $(P \Rightarrow (Q \Rightarrow R))$. They are very different.

e) Statement form

Section 1.3 Q16

Solution:
Any proposition can have two possible values. Therefore, a truth table with n different propositions has 2^n rows. Fixing the first proposition to F left us with $n-1$ free propositions so the number of rows is 2^{n-1}. Further fixing the second value to T leaves $n-2$ free propositions so the number of rows is 2^{n-2}.

Section 1.3 Q18

Solution:

a)

P	Q	$\sim P$	$\sim P \Rightarrow Q$	$P \Rightarrow (\sim P \Rightarrow Q)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

b)

P	Q	R	$P \wedge Q$	$P \vee R$	$(P \wedge Q) \Rightarrow (P \vee R)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	F	T

c)

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \Leftrightarrow Q$	$((P \Rightarrow Q) \Leftrightarrow Q) \Rightarrow P$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

d)

P	Q	$Q \Rightarrow P$	$Q \Rightarrow (Q \Rightarrow P)$	$P \Rightarrow (Q \Rightarrow (Q \Rightarrow P))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T