Math 583: Bridgeland stability

Jarod Alper

Potential topics for presentations

(1) Quiver GIT: Construct the moduli of semistable representations of a quiver using GIT
  - King, *Moduli of representations of finite dimensional algebras.*
(2) Building on (1) (or possibly in addition to (1)), sketch a construction of the moduli space of semistable sheaves using the GIT of quiver representations.
  - Alvarez-Consul and King, *A functorial construction of moduli of sheaves.*
(3) Differential geometry of stable bundles: Present the correspondence between stable vector bundles on a curve of degree 0 and irreducible unitary representations of $\pi_1(C)$. There are many extensions of this result.
  - Narasimhan and Ramanan, *Moduli of vector bundles on a compact Riemann surface.*
  - Kobayashi, *Differential geometry of complex vector bundles.*
(4) Show that the space of stability conditions on $\mathbb{P}^1$ is $\mathbb{C}^2$.
  - Okada, *Stability manifold of $\mathbb{P}^1$.*
(5) Mirror symmetry: Discuss the connection between Bridgeland stability and mirror symmetry. You could focus just on the case of elliptic curves.
  - Douglas, *Dirichlet branes, homological mirror symmetry, and stability.*
  - Bridgeland, *Spaces of stability conditions.*
(6) Moduli spaces of complexes
  - Lieblich, *Moduli of complexes on a proper morphism.*
  - Toda, *Moduli stacks and invariants of semistable objects on K3 surfaces.*
  - Inaba, *Toward a definition of moduli of complexes of coherent sheaves on a projective scheme.*
(7) Bogomolov inequalities: Present a proof of the Bogomolov inequality for smooth projective surfaces. See Theorem 6.14 in Macri and Schmidt's *Lectures on Bridgeland stability* and the references listed there.
(8) Application to the birational geometry of spaces of sheaves on $\mathbb{P}^2$:
  - Coskun and Huizenga, *The birational geometry of the moduli spaces of sheaves on $\mathbb{P}^2$.*
  - Arcara, Bertram, Coskun, and Huizenga, *The minimal model program for the Hilbert scheme of points on $\mathbb{P}^2$ and Bridgeland stability.*
(9) Application to the MMP of sheaves on a K3:
  - Bayer and Macri, *Projectivity and birational geometry of Bridgeland moduli spaces.*
  - Bayer and Macri, *MMP for moduli of sheaves on K3s via wall-crossing.*

- Minamide, Yanagida, and Yoshioka, *Fourier–Mukai transforms and the wall-crossing behavior for Bridgeland's stability conditions*.
- Minamide, Yanagida, and Yoshioka, *Some moduli spaces of Bridgeland's stability conditions*.

(10) Bridgeland stability conditions on your favorite surface: The papers below study Bridgeland stability on particular surfaces. You could present on one of these papers or pick your favorite explicit surface and see what you can say about the space of stability condtions there.
- (Abelian surfaces)
    - Bridgeland, Section 15 of *Stability conditions on K3 surfaces*.
    - Maciocia and Meachan, *Rank 1 Bridgeland stable moduli spaces on a principally polarized abelian surface*.
    - Yanagida and Yoshioka, *Bridgeland's stabilities on abelian surfaces*.
    - Yoshioka, *Bridgeland's stability and the positive cone on the moduli spaces of stable objects on an abelian surface*.
- (Enriques surfaces) Nuer, *Projectivity and birational geometry of moduli spaces of Bridgeland stable objects on an Enriques surface*.
- (Hirzebruch and del Pezzo surfaces) Bertram and Coskun, *The birational geometry of the Hilbert scheme of points on surfaces*.

(11) Bridgeland stability conditions on your favorite quiver.
- Bridgeland, *Stability conditions and the A_2-quiver*.

(12) Polynomial Stability Conditions: Present Bayer's generalization of Bridgeland stability to polynomial Bridgeland stability
- Bayer, *Polynomial Bridgeland stability conditions and the large volume limit*

(13) Bridgeland stability conditions on 3-folds
- See Chapter 9 of (and the references within) Macri and Schmidt, *Lectures on Bridgeland stability*.