## Math 583: Bridgeland stability

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## *Course description:*

The goal of this course is to introduce Bridgeland stability. After discussing in detail stability conditions on an abelian category, we will introduce the notion of a Bridgeland stability condition on a triangulated category. We will show that the set of Bridgeland stability conditions naturally has the structure of a complex manifold. An emphasis will be placed on understanding Bridgeland stability through explicit examples such as representations of quivers and sheaves on a projective variety.

This course will be independent of *Math* 582: *Moduli of vector bundles on a curve*. However, the properties of stability of vector bundles (e.g., the Harder–Narasimhan filtration) will serve as strong motivation for the definition and properties of Bridgeland stability.

## Topics:

- (1) Stability conditions on an abelian category, Harder–Narasimhan filtrations;
- (2) Example of stability conditions on the category of quiver representations;
- (3) Bridgeland stability conditions on a triangulated category;
- (4) The space of stability conditions: its topology and its structure as a complex manifold;
- (5) Bridgeland stability conditions on a smooth projective curve of positive genus;
- (6) Bridgeland stability conditions on the  $A_2$ -quiver;
- (7) Moduli of sheaves on smooth projective surfaces: slope-semistable sheaves and Gieseker semistable sheaves ;
- (8) Existence of stability conditions on smooth projective surfaces and the moduli of Bridgeland semistable objects with respect to a fixed stability condition;
- (9) The wall and chamber decomposition of the space of stability conditions; and
- (10) The space of Bridgeland stability conditions on a K3 surface.

*Prerequisites:* Some familiarity with derived categories.