Math 581: Moduli of vector bundles on a curve

Instructor: Jarod Alper Preferred quarter: Winter Preferred time: MW or WF, anytime

Course description:

Given a smooth, projective curve C of genus g, we will study the moduli space of semistable vector bundles on C of rank r and degree d. The goal of this course will be to establish that such a moduli space exists and to discuss its geometry.

Topics:

- (1) first examples: the moduli of vector bundles on \mathbb{P}^1 (case of g = 0), moduli of vector bundles on an elliptic curve (case of g = 1), the moduli of line bundles on *C* (case of r = 1);
- (2) stability/semi-stability of vector bundles;
- (3) the Harder–Narasimhan and Jordan–Hölder filtrations;
- (4) geometric invariant theory (GIT);
- (5) constructing the moduli space of semistable vector bundles on *C* of fixed rank and degree as a projective variety using GIT;
- (6) first properties of the moduli space: dimension, singularities, normality, universal families;
- (7) further properties: semistable reduction (Langton's theorem), irreducibility, non-emptyness, unirationality, Picard group, the canonical class;
- (8) possible additional topics: rationality of the moduli space, the moduli of sheaves on a projective variety of arbitrary dimension.

Prerequisites: An understanding of basic algebraic geometry.

References:

- J. Le Potier, Lectures on Vector Bundles
- D. Huybrechts and M. Lehn, The Geometry of Moduli Spaces of Sheaves