Math 581B:
Algebraic Groups I

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Course description. The topics of algebraic groups is a rich subject combining both group-theoretic and algebro-geometric-theoretic techniques. Examples include the general linear group GL_N, the special orthogonal group SO_N or the symplectic group Sp_N. The theory of algebraic groups plays an important role in algebraic geometry, representation theory and number theory.

In this course, we will take the functorial approach to the study of linear algebraic groups (more generally, affine group schemes) equivalent to the study of Hopf algebras. The classical view of an algebraic group as a variety will come up as a special case of a smooth algebraic group scheme. Our algebraic approach will be independent (even complementary) to the analytic approach taken in the course on Lie groups.

Prerequisites. Modern Algebra 504/5/6. The course will be suitable for a 2nd year or above graduate student leaning towards an algebra-related field (understood broadly: combinatorics, representation theory, algebraic geometry, algebraic topology). the second year graduate algebra course "Algebraic structures" is desirable but not required.

Topics at a glance:

- Group schemes over an arbitrary base
- Affine group schemes vs Hopf algebras;
- Representations: modules vs. comodules;
- Examples and special cases: abelian group schemes and Cartier duality, étale group schemes, matrix groups, groups of multiplicative type (tori), unipotent groups, nilpotent and solvable groups (i.e. Borel subgroups of semisimple groups);
- Barsotti-Chevalley Theorem on Algebraic Groups;
- Existence of quotients of algebraic groups;
- Detour on descent and algebraic spaces;
- Actions of algebraic groups on schemes and G-torsors;
- Geometric properties: connectedness, irreducibility, smoothness;
- Jordan decompositions;
- Tannaka duality;