SYLLABUS FOR MATH 508, WINTER QUARTER 2016-7: ADVANCED COMMUTATIVE ALGEBRA

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The following is my current plan for the course. We may not have the time to cover all of these topics and we may choose to deviate slightly from the below syllabus.

For each topic, in addition to proving the foundational theorems, there will be strong emphasis on understanding the theory through examples. I will also try to discuss some implications of the main theorems to other mathematical fields such as algebraic geometry and representation theory. We will experiment with the computational software *Macualay2* in order to have a wider array of examples and to enrich our intuition of these algebraic concepts.

(1) Review of basic commutative and homological algebra
(2) Dimension theory of rings: Krull dimension, height of prime ideals, Krull's principal ideal theorem, Krull's height ideal theorem, system of parameters
(3) Flatness: properties of Tor, equational criterion for flatness, local criterion for flatness, flatness of graded modules, relation between flatness and Hilbert polynomials
(4) Completions: Artin–Rees lemma, properties of completion, Hensel's lemma, Cohen's structure theorem
(5) Free resolutions: Koszul complex, regular sequences, minimal free resolutions, projective dimension, graded Betti numbers and Betti tables
(6) Cohen-Macaulay rings and modules: grade, depth, the Auslander-Buchsbaum theorem, definition and properties of Cohen-Macaulay modules, relation between Cohen-Macaulay and flatness
(7) Regularity and normality: regular rings are Cohen-Macaulay, the Ausland-Buchsbaum-Serre theorem, regular rings are UFDs (and thus normal), Serre's normality criterion
(8) Injective hulls and Matlis duality: injective dimension, essential extensions and injective hulls, Matlis duality
(9) Canonical modules and Gorenstein rings: maximal Cohen-Macaulay modules, canonical modules, Gorenstien rings, zero dimensional Gorenstein rings
(10) Local cohomology: definition of local cohomology functors, Grothendieck's vanishing theorem, Grothendieck's local duality theorem