Math 504: Modern Algebra, Fall Quarter 2017
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Midterm Examination
Due: Monday, November 6

Please sign the following statement:

I pledge that my answers on this examination are my own work. I have not discussed this exam with other students and I have not received any assistance of any kind.

Signature: ____________________________

Problem 1.1. Classify all groups of order 385 up to isomorphism.

Problem 1.2. Let $G = \mathrm{GL}_3(\mathbb{C})$ be the group of invertible 3×3 matrices with entries in $\mathbb{C}$. Let $H \subset G$ be the subgroup consisting of diagonal matrices.

(1) Find the centralizer $C_G(H)$ of H in G.
(2) Find the normalizer $N_G(H)$ of H in G.
(3) Determine the quotient $N_G(H)/H$; that is, identify this group with a familiar group.

Problem 1.3. Find *all* possible composition series for the dihedral group D_{12} of order 12.

Problem 1.4. Prove that a group of order p^2q is solvable, where p and q are distinct primes. (Recall from HW Problem 2.2 that a finite group is *solvable* if and only if there exists a composition series with abelian factors.)

Problem 1.5.

(1) Find a Sylow 7-subgroup of $\mathrm{GL}_3(\mathbb{F}_2)$.
(2) Show that $\mathrm{GL}_3(\mathbb{F}_2)$ is isomorphic to the subgroup of S_7 generated by the permutations (1234567) and $(15)(23)$.

   Hint: Consider the action of $\mathrm{GL}_3(\mathbb{F}_2)$ on the set $\mathbb{F}_2^3 \setminus 0$ containing 7 elements and the induced homomorphisms $\mathrm{GL}_3(\mathbb{F}_2) \to S_7$.

Problem 1.6. Let k be a field.

(1) Show that the ideal $(xy - zw) \subset k[x, y, z, w]$ is prime.
(2) Show that the element $x \in k[x, y, z, w]/(xy - zw)$ is irreducible but not prime.