SYLLABUS FOR MATH 504, FALL QUARTER 2017-8: MODERN ALGEBRA

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The following is the rough outline of the course. As the course progresses, a brief summary of what is covered in the lectures will be posted on the website.

(1) Group theory
- Review of the basics.
- Examples of groups: free groups, direct and semi-direct products, extensions, finitely generated abelian groups.
- Simple groups, composition series, Jordan–Hölder theorem, solvable and nilpotent groups, upper central and derived series.
- Sylow theorems.

(2) Rings
- Review of the basics.
- Examples of rings: polynomials rings, matrix rings, group rings, invariant rings, elementary symmetric polynomials.
- Properties of rings: Euclidean, Principal ideal and Unique factorization domains, Gauss's lemma, Eisenstein's criterion.
- Simple and semi-simple rings, Artin–Wedderburn theorem.

(3) Modules
- Review of the basics.
- Structure theorem for modules over a PID.
- Noetherian and Artinian rings and modules.
- Simple modules, composition series, and Jordan–Hölder theorem for modules.
- Applications to linear algebra (characteristic and minimal polynomials, Cayley–Hamilton theorem, rational canonical form, canonical Jordan form).
- Tensor products.