Math 504: Modern Algebra, Fall Quarter 2017
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Homework 3
Due: Monday, October 16

Problem 3.1. Show that two permutations in S_n are conjugate if and only if they have the same cycle type.

Problem 3.2. Suppose $G \subset \mathrm{SO}_3(\mathbb{R})$ is a subgroup of order 12. As in lecture, let $\Omega \subset S^2$ be the set of points on the sphere fixed by a non-identity element of G. Suppose that the action of G on Ω has 3 orbits of size $6, 4$ and 4.[1] Show that G is the symmetry group of a tetrahedron.

Problem 3.3. Let p be a prime. Find a p-Sylow subgroup of $\mathrm{GL}_n(\mathbb{F}_p)$. How many p-Sylows are there?

Problem 3.4. Find all 2, 3 and 5-Sylow subgroups of A_5.

Problem 3.5. Classify up to isomorphism finite groups of order 18.

Problem 3.6. Classify up to isomorphism finite groups of order 20.

[1] In the notation from lecture, this means that $(n_1, n_2, n_3) = (6, 4, 4)$ and $(r_1, r_2, r_3) = (2, 3, 3)$, where n_i is the size of the ith orbit and r_i is the order of the stabilizer of an element in the ith orbit.