Definition 1. Let $\{0,1\}*$ be the set of all binary strings of finite length.
Definition 2. A language over $\{0,1\}$ is a subset $A \subset \{0,1\}*$.
Example 1. $EVEN = \{n >= 0 \in \mathbb{Z}$ *written in binary* $|n$ *is even*$\}$, *then*
$n = \{0, 00, 10, 000, 010, 100, 110, \dots\}$.
Example 2. $PRIME = \{p >= 2 \in \mathbb{Z}$ *written in binary* $|p$ *is prime*$\}$, *then*
$p = \{10, 11, 101, \dots\}$.

More generally, can consider any set Σ (which we call alphabet) and set $\Sigma*$, the set of all string of Σ of finite length.
Definition 3. A language over Σ is a subset $A \subset \Sigma*$.
Example 3. $\Sigma = \{0, 1, 2\}$.
Example 4. $\Sigma = \mathbb{Z}/n := \{0, 1, \dots, n-1\}$.
Example 5. $\Sigma = \mathbb{Z}$.
Example 6. $\Sigma = \mathbb{R}$.
Goal 1. Given a language $A \subset \Sigma*$, we would like a computational model for determining if a given string $w \in \Sigma*$ is in A.

Turing Machines:
Definition 4 (Conceptual)**.** A **Turing Machine** is a finite state machine (which has a finite set (which we will call states) and a finite set of rules which govern the action depending on the current state and what the head reads from the tape).
The action is either

a print a symbol form Σ.

b moves to left or right.

c move to a different state.

d terminate with "accept" or "reject."

Definition 5 (Precise)**.** A **Turing Machine** is a 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$

1. Q is a finite set (of states).
2. Σ is the input alphabet (not containing the blank symbol).
3. Γ is tape alphabet (containing the blank, $\Sigma \subset \Gamma$).
4. δ is $Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$
5. $q_0 \in Q$ is the start state.
6. $q_{\text{accept}} \in Q$ is the accept state.
7. $q_{\text{reject}} \in Q$ is the reject state.

Definition 6. Let $A \subset \Sigma*$. We say a turing maching M **recognizes** A if for any $w = w_1, w_2, \dots, w_n \in \Sigma*$, then M accepts $w \Leftrightarrow w \in A$.
Definition 7. A language is **recognizable** if there exists a turing machine which recognizes it.