Math 480

Review

Let k = field, $VP = \{\{f_n\}$ p-computable$\}$

If $\{f_n\}$ s.t. the coefficient of f_n can be computed in poly time, then $\{f_n\}$ is p-computable. $\rightarrow$ $VNP = \{\{f_n\}$ p-definable$\}$.

We say $\{f_n\}$ is p-defined if # variables & degree of f_n are poly bounded. and either (a) $\exists$ p-computable $\{g_n\}$.

$$f_n = \sum_{e_1, \cdots, e_n \in \{0,1\}} g_n(e_1, \cdots, e_n) x_1^{e_1} \cdots x_n^{e_n}$$

or

(b) $\{f_n\}$ is p-projected or sth. of form (a).

In HW: If $\{g'_n\}$ is p-computable $\Rightarrow f'_n = \sum_{|i|=d} g'_n(i_1, \cdots, i_j) x_1^{i_1} \cdots x_d^{i_j}$ p-definable

Suppose $k = \mathbb{Z}/2$ (or more generally any field of char $= 2$)

$\det_n = \mathrm{perm}_n \in k[x_{11}, \cdots, x_{nn}] \Rightarrow \mathrm{perm}_n \in VP$

Q: Is $\{\mathrm{perm}_n\} \in VP$?

prop: $\{\mathrm{perm}_n\} \in VNP$

proof: Claim $\exists$ p-computable $g_n(x_{11}, \cdots, x_{nn})$ n^2 variables s.t.

$g_n(e_{11}, \cdots, e_{nn})$ = coeff of $x_{11}^{e_{11}} \cdots x_{nn}^{e_{nn}}$ is perm_n.

$= \begin{cases} 1 & \text{if } (e_{ij}) \text{ is a matrix with precisely one 1 in every row and column and 0's elsewhere} \\ 0 & \text{o/w} \end{cases}$

Because then $\mathrm{perm}_n = \sum_{e_{ij} \in \{0,1\}} g_n(e_{11}, \cdots, e_{nn}) x_{11}^{e_{11}} \cdots x_{nn}^{e_{nn}}$

Sol: $g_n(x_{11}, \cdots, x_{nn}) = \prod_{j=1}^{n} \left(\sum_{i=1}^{n} x_{ij} \right) \prod_{\substack{i \neq k \\ j = m}} (1 - x_{ij} x_{km})$

$$\begin{pmatrix} e_{11} & \cdots & e_{1n} \\ e_{21} & \cdots & e_{2n} \\ \vdots & & \vdots \\ e_{n1} & \cdots & e_{nn} \end{pmatrix}$$

$\ne 0$ for jth col $= 0$.