Recap: For languages $A \subset \{0,1\}^*$, there are two language claess: P and NP
- $P \subset NP \subset$ {recognizable}
- {decidable} $\subset$ {recognizable}
- A_{TM} is recognizable but not decidable

DEF A (undirected) graph is a finite set of vertices with edges connecting some of the vertices.
More precisely, a graph $G = (V, E)$ where V is a **finite** set and $E \subset V \times V$

DEF A directed graph is a finite set $G = (V, E)$ where V is a **finite** set and $E \subset V \times V$
Notation: given an edge $e = (v_1, v_2)$, the source of e, $s(e) := v_1$; the target of e, $t(e) := v_2$.

DEF A labelled graph is a finite set of vertices with edges connecting some of the vertices s.t. each edge is labelled.

DEF A path in G is a seqeunce of edges $e_1, ..., e_n$ with $t(e_i) = s(e_{i+1})$ for $i = 1, .., n-1$

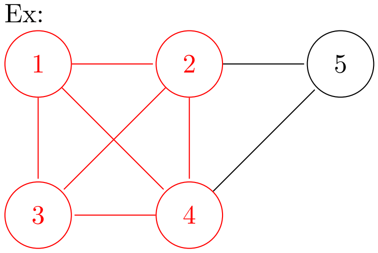
DEF A Hamiltonian path in G is a path which passes through each vertex exactly once.

Assume that a graph can be encoded as a binary string in $\{0,1\}^*$.
Then we can define a language, **HAMPATH**={directed graphs G chich have a Hamiltonian path}
Let $n = |V| = $ #vertices in G, then the numer of possible paths is $n!$

proposition There is a TM deciding HAMPATH with running time $O((\sqrt{n})!)$ which is worse than $O(2^n)$.

proposition HAMPATH $\in$ NP
proof: We can design a NDTM as follow:
(1) Input directed graph $G = (V, E)$. Let $n = |V|$.
(2) Assume $V = \{1, ..., n\}$
(3) For each i_1 from 1 to n:
(4) As j goes from 1 to n
(5) brach over every edge with $s(e) = i_j$, set $i_{j+1} = t(e)$.
(6) Go back to (4)
(7) check if $(i_1, ..., i_n)$ is a Hamiltonian path. If yes, *accpet*.

DEF A k-Clique of a graph $G = (V, E)$ is a subset $V' \subset V$ of k vertices s.t. G contains every edge between.
Ex:



Then we define the language: **CLIQUE**=$\{(G, k)|$ G has a k-clique$\}$.

proposition CLIQUE $\in$ NP.

Quesiton to consider: Are HAMPATH and CLIQUE in P?