Math 480A: Algebraic Complexity Theory, Spring Quarter 2019
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Homework 1
Due: Friday, April 12

**Problem 1.1.** Sipser 3.1 (all editions)

**Problem 1.2.** Design a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ that decides the language $\text{EVEN} = \{n \in \mathbb{Z} \text{ written in binary} \mid n \geq 0 \text{ is even}\}$.
*You should provide all of the details.*

**Problem 1.3.** Design a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ that decides the language $\text{TRIPLE} = \{n \in \mathbb{Z} \text{ written in binary} \mid n \geq 0 \text{ is divisible by } 3\}$.
*You should provide all of the details.*

**Problem 1.4.** Sipser 3.6 (all editions)

**Problem 1.5.** Sipser 3.7 (all editions)

**Problem 1.6.** Describe informally a non-deterministic Turing machine that recognizes the language $A$ consisting of strings of 0's and 1's which contain two disjoint copies of the same substring of consecutive 0's and 1's of length 10. Precisely

$$A = \{w_1 w_2 w_3 w_2 w_4 \mid w_1, w_2, w_3, w_4 \in \{0,1\}^* \text{ with } |w_2| = 10\}.$$

*You should provide some but not all of the details to give the general idea. You may find it convenient to use a Turing machine with multiple tapes.*

**Problem 1.7.** Consider homogeneous polynomials $f(x_1, \ldots, x_n)$ of degree $n$ whose coefficients are 0 or 1.

(a) Show that such polynomials can be encoded as a binary string.
(b) Describe informally a non-deterministic Turing machine that recognizes the language $A$ consisting of polynomials $f(x_1, \ldots, x_n)$ of degree $n$ which have a non-trivial zero modulo 2 (that is, there exists $(a_1, \ldots, a_n) \in \{0,1\}^n$ not all zero with $f(a_1, \ldots, a_n) \equiv 0 \mod 2$).

*As with the previous problem, you should provide some but not all of the details.*

**Problem 1.8** (Sipser 3.16 in 3rd ed)**.** Show that the collection of Turing-recognizable languages is closed under the operation of

(1) union
(2) concatenation
(3) star
(4) intersection
(5) homomorphism.

**Problem 1.9** (Sipser 3.15 in 3rd ed)**.** Show that the collection of decidable languages is closed under the operation of

(1) union
(2) concatenation
(3) star
(4) complementation

(5) intersection.

**Problem 1.10** (Sipser 3.11 in 3rd ed)**.** A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.