Math 427A: Complex Analysis, Fall Quarter 2018
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Homework 3
Due: Monday, October 22

Problem 3.1. Let $E \subset \mathbb{C}$ be an open set and $f: E \to \mathbb{C}$ be a function. If f is differentiable at a point $z \in E$, show that f is also continuous at z.

Problem 3.2. Prove the product formula: if f and g are complex functions that are differentiable at z, then fg is differentiable at z with derivative $(fg)'(z) = f'(z)g(z) + f(z)g'(z)$.

Problem 3.3. Use Problem 3.2 and induction to show that

$$\frac{d(z^n)}{dz} = nz^{n-1}.$$

Problem 3.4. Taylor 2.2.8

Problem 3.5. Taylor 2.2.11

Problem 3.6. Taylor 2.2.12

Problem 3.7. Taylor 2.2.13

Problem 3.8. Taylor 2.2.15

Problem 3.9. For each of the following following functions of z, express the function in the form $u(x, y) + iv(x, y)$ where $z = x + iy$:

(a) $z^3 + \overline{z}^3$
(b) $z^2 e^z$
(c) $\cos(z)$

Problem 3.10. For each Part (a)–(c) of Problem 3.10, use the Cauchy–Riemann equations to determine if the function is analytic.