Please complete ONE of the following options.

Option 10.1. Finish the proof of Galois' criterion by showing that if $f(x) \in K[x]$ is a polynomial with splitting field L such that $\mathrm{Gal}(L/K)$ is solvable, then $f(x)$ is solvable by radicals. (We proved the other direction in lecture.)

You may follow an exposition from a textbook or online source, but please indicate what source you are using and write up the argument in your own words.

Option 10.2.
(a) Show that S_n is not solvable for $n \geq 5$.
(b) Show that there exists a polynomial $f(x) \in \mathbb{Q}[x]$ of degree 5 such that the Galois group of the splitting field of $f(x)$ over $\mathbb{Q}$ is isomorphic to S_5.

You may follow an exposition from a textbook or online source, but please indicate what source you are using and write up the argument in your own words.

Option 10.3. Write a summary of a mathematical article with content relating to Galois theory. Your summary should be roughly 500 words.

You can choose an article giving an expository account of mathematical research, an article about a particular mathematician's contribution to the field, or an article about real-world applications of mathematics. It's completely your choice as long as it is tangentially related to Galois theory. If you would like a suggestion, I recommend the mathematical articles in Quanta Magazine. They have several articles about polynomials (see https://www.quantamagazine.org/tag/polynomials/). In particular, the recent articles addressing advances on Hilbert's 12th and 13th problem might be good options:
- https://www.quantamagazine.org/mathematicians-probe-unsolved-hilbert-polynomial-problem-20210114/
- https://www.quantamagazine.org/mathematicians-find-polynomial-building-blocks-hilbert-sought-20210525/

Try to highlight the connections to Galois theory and the content we covered in the course. Please cite your sources that you use in your summary.