Math 403B: Introduction to Modern Algebra, Winter Quarter 2018
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Homework 4
Due: Wednesday, February 7

Problem 4.1. Judson 17.4.2 (a), (d), (f)

Problem 4.2. Judson 17.4.4

Problem 4.3. Judson 17.4.8

Problem 4.4. Judson 17.4.9

Problem 4.5. Find all irreducible polynomials of degree 4 in $\mathbb{Z}/2[x]$.

Problem 4.6. In this exercise, you will prove directly, without using Eisenstein's criteria, that $\mathbb{Q}[x]$ contains an irreducible polynomial of every possible positive degree. Let n be a positive integer greater than one. Prove that $x^n - 2$ is irreducible in $\mathbb{Q}[x]$ following these steps:

(a) Suppose that $x^n - 2 = g(x)h(x)$ with $g(x), h(x) \in \mathbb{Z}[x]$ with degrees k and l, both strictly less than n. Show that 2 divides the constant term of g or h but not both.
(b) Make a choice in (a) by assuming that 2 divides the constant term of g but not of h. Argue that 2 divides the degree 1 coefficient of g.
(c) Arguing similarly, show that 2 divides the degree 2 coefficient of g, the degree 3 coefficient of g and so on. Obtain a contradiction and conclude that $x^n - 2$ is irreducible.

Problem 4.7. Judson 17.4.14

Problem 4.8. Judson 17.4.15

Problem 4.9. Judson 17.4.18