Math 403B: Introduction to Modern Algebra, Winter Quarter 2018
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Homework 3
Due: Monday, January 29

Problem 3.1. If R is a commutative ring and $I \subset R$ is an ideal, show that there is a bijective correspondence between ideals in R/I and ideals in R containing I.

Problem 3.2. Judson 16.6.34

Problem 3.3. Let $p \in \mathbb{Z}$ be a positive prime integer. Describe all the maximal ideals in the ring $\mathbb{Z}_{(p)}$ of integers localized at p.

Problem 3.4. Judson 16.6.37

Problem 3.5. Judson 16.6.40

Problem 3.6. Let $\phi\colon R \to S$ be a ring homomorphism.

(a) Show that if $\mathfrak{p} \subset S$ is a prime ideal, then $\phi^{-1}(\mathfrak{p}) \subset R$ is a prime ideal.
(b) Show that the conclusion of (a) is false if the word "prime" is replaced by "maximal."

Problem 3.7. In the ring of Gaussian integers $\mathbb{Z}[i]$, consider the principal ideal $(1 + 2i)$. Draw a picture of all of this ideal sitting inside the lattice $\mathbb{Z}[i] \subset \mathbb{C}$ inside the complex numbers. Explain why $\{0, i, 2i, -1 + i, -1 + 2i\}$ is a complete set of congruence class representatives for the ring $\mathbb{Z}[i]/(1 + 2i)$. Show this quotient ring is isomorphic to $\mathbb{Z}/5$ by calculating addition and multiplication tables for these representatives.

Problem 3.8. Judson 17.4.3

Problem 3.9. Judson 17.4.5

Problem 3.10. Let R be a commutative ring and define $R[x, y]$ to be the ring of polynomials over R in the variables x and y. Show that there is an isomorphism of rings

$$R[x, y] \cong R[x][y].$$