1. a. If some dogs bark then all cats meow.

b. If no dogs bark then some cats do not meow.

c. all cats meow and no dogs bark.

2. $\underset{\text{even}}{2n} + \underset{\text{even}}{2m} = \underset{\text{even}}{2(n+m)}$

$\underset{\text{odd}}{2n+1} + \underset{\text{odd}}{2m+1} = \underset{\text{even}}{2(n+m+1)}$

$\underset{\text{even}}{2n} + \underset{\text{odd}}{2m+1} = \underset{\text{odd}}{2(n+m)+1}$

3. a. $P \to (Q \to R)$

$P \to (\sim Q \vee R)$

$\sim P \vee (\sim Q \vee R)$

b. $\sim(\sim P \wedge \sim Q)$

c. we can replace $\to$ with $\sim$, $\vee$, and $\wedge$ and we can replace $\vee$ with $\sim$ and $\wedge$.

4. a.

P	$\sim P$	$P \uparrow P$
T	F	F
F	T	T

b.

P	Q	$P \vee Q$	$P \uparrow P$	$Q \uparrow Q$	$(P \uparrow P) \uparrow (Q \uparrow Q)$
T	F	T	F	T	T
F	F	F	T	T	F
T	T	T	F	F	T
F	T	T	T	F	T

c.

P	Q	$P \wedge Q$	$P \uparrow Q$	$(P \uparrow Q) \uparrow (P \uparrow Q)$
T	F	F	T	F
T	T	T	F	T
F	F	F	T	F
F	T	F	T	F

d. $P \to Q \equiv P \uparrow (Q \uparrow Q)$

P	Q	$P \to Q$	$P \uparrow (Q \uparrow Q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

5. $S_1 \leftrightarrow S_2$ is a statement that can be true or false, but $S_1 \equiv S_2$ is an assertion that $S_1 \leftrightarrow S_2$ is a tautology.

6. $((P \wedge Q) \vee (P \wedge \sim Q)) \wedge (R \vee \sim R) \equiv$

$((P \wedge Q) \vee (P \wedge \sim Q)) \equiv \boxed{P}$

7. $A = \{n \mid n = 2Y-2,\ Y \in \mathbb{N}\}$

$B = \{n \mid n = (Y-1)^2+1,\ Y \in \mathbb{N}\}$

$C = \{n \mid n = 4Y-3,\ Y \in \mathbb{N}\}$

$D = \{n \mid \frac{1}{n} \in \mathbb{N}\}$

$E = \{n \mid n \in \mathbb{N} \vee n = \text{lemon}\}$

8. $\exists x P$ is true, $\forall x P$ is false.

a. $T \to F$ is <u>False</u>

b. $F \to T$ is <u>true</u>

9. p is prime $\leftrightarrow p \in \mathbb{N} \wedge p \geq 1 \wedge (a, b \in \mathbb{N} \wedge a \cdot b = p \to a = p \vee b = p)$

10. a. True

b. False

c. True

d. True

e. False

f. True

g. True

h. True