Algebra 2, Semester 1 2015
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Tutorial 9
Due: Friday, May 15

Tutorial 9.1. Show that the polynomial $x^5 - 4x + 2 \in \mathbb{Q}[x]$ is not solvable by radicals.

Tutorial 9.2. Recall that the *discriminant* of a polynomial

$$f(x) = \prod_{i=1}^{n} (x - \alpha_i) \in \mathbb{Q}[x]$$

is

$$\Delta(f) = \prod_{1 \le i < j \le n} (\alpha_i - \alpha_j)^2.$$

Show that $\Delta(f)$ is a square in $\mathbb{Q}$ if and only if Gal(f) is contained in A_n. (*Hint*: Let $\delta(f) = \sqrt{\Delta(f)} = \prod_{1 \le i < j \le n} (\alpha_i - \alpha_j)$ and consider the action of Gal(f) on $\delta(f)$.)

Tutorial 9.3. Let $f(x) \in \mathbb{Q}[x]$ be an irreducible quartic with roots $\alpha_1, \ldots, \alpha_4$. Show that the cubic polynomial $r(x)$ with roots

$$(\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4),\ (\alpha_1 + \alpha_3)(\alpha_2 + \alpha_4),\ (\alpha_1 + \alpha_4)(\alpha_2 + \alpha_3)$$

is irreducible over $\mathbb{Q}$ if and only if Gal(f) is S_4 or A_4.

Tutorial 9.4. Using questions 2 and 3, determine the Galois groups of the following quartics:

(a) $x^4 + x + 1$
(b) $x^4 + 4x + 3$

(*Hint*: Recall that given a quartic of the form $f(x) = x^4 + a_3 x + a_4$, $\Delta(f) = -27a_3^4 + 256a_4^3$ and $r(x) = x^3 - 4a_4 x + a_3^2$.)