Algebra 2, Semester 1 2015
Jarod Alper
Tutorial 2

Tutorial 2.1. Prove that $\mathbb{R}[x]/(x^2+1) \cong \mathbb{C}$.

Tutorial 2.2. Use Lagrange's method to solve

$$x^4 - 16 = 0.$$

Tutorial 2.3. Find a polynomial $f(\alpha_1, \alpha_2, \alpha_3)$ whose orbit under S_3 consists of only two elements.

Tutorial 2.4. Consider the polynomial

$$\begin{aligned} f(\alpha_1, \dots, \alpha_5) = &(\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_4 + \alpha_4\alpha_5 + \alpha_5\alpha_1 \\ &- \alpha_1\alpha_3 - \alpha_2\alpha_4 - \alpha_3\alpha_5 - \alpha_4\alpha_1 - \alpha_5\alpha_2)^2 \end{aligned}$$

(1) Show that the orbit of $f(\alpha_1, \cdots, \alpha_5)$ under the action of S_5 consists of 6 elements.
(2) Let
$$x^5 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$$
be a quintic equation (i.e. degree 5 equation) with five distinct roots $\alpha_1, \dots, \alpha_5$. Show that the six values $f_1(\alpha_1, \dots, \alpha_5)$, $\dots$, $f_6(\alpha_1, \dots, \alpha_5)$ (where the functions $f_1, \dots, f_6$ denote the orbit of f) are the solutions to a sextic equation (i.e. degree 6 equation).
(3) Why does this not help in solving the quintic?

Tutorial 2.5. Let A be a ring and $I \subset A$ be an ideal. Let $\phi : A \to A/I$ be the canonical ring homomorphism. Prove the following assertion: If $\psi : A \to B$ is an ring homomorphism with $I \subset \ker(\psi)$, then there exists a unique ring homomorphism $\lambda : A/I \to B$ such that $\psi = \lambda \circ \phi$.

In other words, you need to show that given such a ψ, there is a unique dotted arrow filling in the diagram