Algebra 2, Semester 1 2015
Jarod Alper
Homework 6
Due: Wednesday, April 1

Problem 6.1. Prove that there exists an inclusion of fields $\mathbb{F}_{p^a} \subseteq \mathbb{F}_{p^b}$ if and only if $a|b$.

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Problem 6.2. Let p be a prime and $q = p^n$. Consider the map

$$\begin{aligned} \sigma \colon \mathbb{F}_q &\to \mathbb{F}_q \\ x &\mapsto x^p. \end{aligned}$$

(1) Show that σ is a well-defined homomorphism of fields.
(2) Using that there is a field inclusion $\mathbb{F}_p \subseteq \mathbb{F}_q$, show that σ restricts to the identity homomorphism $\mathbb{F}_p \to \mathbb{F}_p$.
(3) Show that $\sigma \colon \mathbb{F}_q \to \mathbb{F}_q$ is an isomorphism.
(4) Show that the set of elements fixed by σ is precisely $\mathbb{F}_p$; in other words, show that

$$\mathbb{F}_p \cong \{x \in \mathbb{F}_q \mid \sigma(x) = x\}.$$

Problem 6.3. Recall that a field extension $K \subseteq L$ is *separable* if for every $\alpha \in L$, the minimal polynomial of α over K has no multiple roots. Show directly from this definition that any degree two field extension $\mathbb{Q} \subset K$ is separable.