Algebra 2, Semester 1 2015
Jarod Alper
Homework 5
Due: Monday, March 23

Problem 5.1. Determine the splitting fields $\mathbb{Q} \subseteq K$ of the following polynomials defined over $\mathbb{Q}$ and compute the degree $|K : \mathbb{Q}|$.
(a) $f(x) = x^3 - 2$.
(b) $f(x) = x^4 - 3$.
(c) $f(x) = x^4 - 2x^2 - 3$.
(d) $f(x) = x^9 - 1$.

Problem 5.2. Show that any field extension $K \subseteq L$ of degree 2 is normal.

Let $K \subseteq L$ be a field extension. Recall that we say $\alpha \in L$ is *separable over* K if the minimal polynomial of α over L has no multiple roots. We say that $K \subseteq L$ is a *separable* field extension if every element $\alpha \in L$ is separable over K. You may use freely the following two properties (which will be proved next week):

- If the characteristic of K is zero, then $K \subseteq L$ is separable.
- If the characteristic of K is p and every element of K has a pth root, then $K \subseteq L$ is separable.

Problem 5.3. For each of the following field extensions, determine whether it is normal and whether it is separable.
(a) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{-5})$.
(b) $\mathbb{Q}(i) \subseteq \mathbb{Q}(\sqrt[3]{2}, i)$.
(c) $\mathbb{F}_p \subseteq \mathbb{F}_{p^n}$ where p is a prime.
(d) $\mathbb{F}_p(x^p) \subseteq \mathbb{F}_p(x)$ where p is a prime.