Algebra 2, Semester 1 2015
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Homework 1
Due: Monday, February 23

Problem 1.1. Find the solution to the following cubic equations using the method discussed in class:
(a) $x^3 - 3x + 2 = 0$.
(b) $x^3 + 3x - 36 = 0$. Which solutions are real? rational?

Problem 1.2. Let $f(x) = x^5 + x^4 + x^3 + x^2 + x - 5$. Notice that $f(1) = 0$. Find the polynomial $g(x)$ such that

$$f(x) = (x - 1)g(x).$$

Problem 1.3.
(a) Express $x_1^4 + x_2^4 + x_3^4 + x_4^4$ as a polynomial in terms of the elementary symmetric functions s_1, s_2, s_3, s_4.
(b) Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ denote the complex roots of the polynomial

$$x^4 + x^3 + 2x^2 + 3x + 5.$$

Determine the number $\alpha_1^4 + \alpha_2^4 + \alpha_3^4 + \alpha_4^4$.

Hint: Do not actually solve for the roots α_i explicitly!

Problem 1.4. Let $f(x) = x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$ be a polynomial with roots $\alpha_1, \ldots, \alpha_n$. The *discriminant* of $f(x)$ is defined as

$$\Delta = \prod_{i<j} (\alpha_i - \alpha_j)^2.$$

(a) Prove that this is a symmetric function.
(b) If $f(x) = x^3 + a_2 x + a_3$, express the discriminant Δ in terms of coefficients a_2, a_3.