55. Conditioned on getting the first typist the probability of getting $k$ errors is $e^{-3}(3)^k/k!$. Conditioned on getting the second the probability of getting $k$ errors is $e^{-4.2}(4.2)^k/k!$. Let $X$ be the number of errors, let $T_1$ be the event that the first typist types the paper and let $T_2$ be the event that the second typist types the paper. Then

$$P(X = 0) = P\left(\{X = 0\}\mid T_1\right) + P\left(\{X = 0\}\mid T_2\right)$$

$$= P(T_1)P\left(\{X = 0\}\mid T_1\right) + P(T_2)P\left(\{X = 0\}\mid T_2\right)$$

$$= .5(e^{-3}) + .5(e^{-4.2})$$

$$\approx .0325.$$

57. The probability that a Poisson random variable $3$ has value 0, 1 and 2 is approximately 0.050, 0.149 and 0.224 respectively. Thus the probability of 3 or more accidents is about $1 - (.05 + .149 + .224) = .577$. When we condition on at least one accident we get

$$P(X \geq 3 \mid X \geq 1) = \frac{P(X \geq 3 \text{ and } X \geq 1)}{P(X \geq 1)}$$

$$\approx .606$$

58. (a) $P(\text{Bin}(8, .1) = 2) \approx .149$ while $P(\text{Poisson}(.8) = 2) \approx .144$.  
(b) $P(\text{Bin}(10, .95) = 9) \approx .315$ while $P(\text{Poisson}(9.5) = 9) \approx .130$.  
(c) $P(\text{Bin}(10, .1) = 0) \approx .349$ while $P(\text{Poisson}(1) = 0) \approx .368$.  
(d) $P(\text{Bin}(9, .2) = 4) \approx .066$ while $P(\text{Poisson}(1.8) = 4) \approx .072$.  

59. To calculate the approximate probability we use the approximate Poisson distribution with parameter $(50)\left(\frac{1}{100}\right) = .5$. We get that the probability of 0 and 1 wins to be about .607 and .303 respectively. Thus the probability of winning
(a) at least once is about $1 - .607 = .393$.
(b) exactly once is about .303.
(c) at least twice is about $1 - .607 - .303 = .090$.

To calculate the actual probabilities we would use the $\text{Bin}(50, .01)$ distribution. We get that the probability of 0 and 1 wins to be about .605 and .306 respectively. Thus the probability of winning

(a) at least once is about $1 - .605 = .395$.
(b) exactly once is about .306.
(c) at least twice is about $1 - .605 - .306 = .089$.

61. Let $X$ be the number of full houses that you are dealt. $X$ is a $\text{Bin}(1000, .0014)$ random variable. We can approximate it by a Poisson variable with parameter 1.4.

\[
P(X \geq 2) = 1 - P(X = 0) - P(X = 1) \\
= 1 - (.986)^{1000} - \binom{1000}{1} (.986)^{999}(.014)^1 \\
\approx .4083
\]

while the Poisson approximation is

\[
P(X \geq 2) = 1 - P(X = 0) - P(X = 1) \\
\approx 1 - e^{-1.4} (1.4)^0 - e^{-1.4} (1.4)^1 / 1! \\
\approx .4082.
\]

64. (a) Let $X$ be the number of people who commit suicide in a month. $X$ is a $\text{Bin}(400, 0.00001)$ random variable. We can approximate it by a Poisson variable with parameter 4.

\[
P(X \geq 8) = 1 - \sum_{k=0}^{7} \binom{400000}{k} (.99999)^{400000-k} (.00001)^k \approx .948866
\]

while the Poisson approximation is

\[
P(X \geq 8) \approx \sum_{k=0}^{7} e^{-4} (4)^k / k! \approx .948867.
\]
(b) Let $Y$ be the number of months with at least 8 suicides. Then

$P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1)$

$\approx 1 - (.949)^{12} - 12(.051)(.949)^{11}$

$\approx .123$ 

(c) Let $Z$ be the first month with at least 8 suicides. $Z$ is negative binomial with $p = .051133$. The probability mass function of $Z$ is

$P(Z = k) = (.948867)^{k-1}(.051133)$.

65. Let $Y$ be the number of people with the disease. The distribution of $Y$ is Bin(500, .001). We can approximate $Y$ by a Poisson distribution with parameter $\lambda = .5$. Then

(a) The probability that there is at least one person with the disease is

$P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-5}(.5)^{0}/0! \approx .3935$.

(b) The probability that there is more than one person with the disease is

$P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1)$

$= 1 - e^{-5}(.5)^{0}/0! - e^{-5}(.5)^{1}/1!$

$\approx .2293$.

(c) Let $Z$ be the number of people other than Jones who have the disease. $Z$ is a Bin(499, .001) random variable which we can approximate with a Poisson random variable with $\lambda = 4.99$. If there are more than one person with the disease then $Z \geq 1$.

$P(Z \geq 1) = 1 - P(Z = 0) = 1 - e^{-4.99}(.499)^{0}/0! \approx .3935$

68. Let $X_i$ be the number antiballistic missiles that hit the $i$th missile. Let $Y$ be the number of missiles that get destroyed by an antiballistic missile. Thus $Y$ is the number of $i$ such that $X_i > 0$. Using the Poisson paradigm we get that each missile independently has a probability of about

$P(X = 0) = e^{-5}5^{0}/0! \approx .99326$.
of being not hit. Since the events are independent and have the same distribution

\[ P(Y = 10) = P\left( \cap_{i=1}^{10} \{ X_i > 0 \} \right) \]
\[ = \prod_{i=1}^{10} \left( 1 - P(X_i = 0) \right) \]
\[ \approx (1 - e^{-5})^{10} \approx 0.065373. \]

75. Let \( X \) denote the number of tails that occur before a heads occurs for the tenth time. The distribution of \( X + 1 \) is negative binomial with parameters 10 and .5.

\[ P(X + 1 = k) = \binom{k-1}{9} .5^{k-10} .5^{10} \]

or

\[ P(X = k) = \binom{k}{9} .5^{k+1}. \]

80. (a) The probability that both number the player selects are among the 20 that the casino selects is

\[ \alpha = \frac{20}{80} \frac{19}{79}. \]

The fair payoff \( f \) is the value that makes the expected winnings equal to 0.

\[ E(W) = f \alpha + (-1)(1 - \alpha) = 0 \]

or

\[ f = (1 - \alpha)/\alpha \approx 15.63. \]

(b) There are \( \binom{80}{20} \) total number of choices for how the casino can choose the 20 numbers. If the player chooses \( n \) and \( k \) of the \( n \) are among the numbers the casino chose then \( n - k \) are not. There are \( \binom{n}{k} \) ways to choose \( k \) out of the \( n \) (that the player chose) and \( \binom{80-n}{20-k} \) to choose \( 20 - k \) out of \( 80 - n \) (that the player did not choose). Thus the probability of choosing \( k \) out of \( n \) is

\[ P_{n,k} = \frac{\binom{n}{k} \binom{80-n}{20-k}}{\binom{80}{20}}. \]
(c) Let \( W \) be the winnings and \( X \) be the number of balls selected correctly. Let \( W'(X) \) be the winnings if \( X \) balls are selected. (If \( s \) is an element of our state space (a selection of 20 numbers) then \( W(s) = W'(X(s)) \).)

\[
E(W) = \sum_{k=0}^{10} P(X = k)W'(k) = \sum_{k=0}^{10} P_{10,k}W'(k)
\]

\[
= (-1)\sum_{k=0}^{4} P_{10,k} + \sum_{k=5}^{10} P_{10,k}W'(k)
\]

\[
\approx (-1)(\sum_{k=0}^{4} P_{10,k}) + (1)P_{10,5} + (17)P_{10,6} + (179)P_{10,7}
\]

\[
+ (1299)P_{10,8} + (2599)P_{10,9} + (24999)P_{10,10}
\]

\[
\approx -.2057
\]

82. Let \( X \) be the number of transistors in a lot that are sampled and defective. The distribution of \( X \) is Bin(4, .1). The lot is rejected if \( X \geq 1 \).

\[
P(X \geq 1) = 1 - P(X = 0) = 1 - (.9)^4 = .3439.
\]