20. A gambling book recommends the following “winning strategy” for the game of roulette. It recommends that a gambler bet $1 on red. If red appears (which has probability $\frac{18}{38}$), then the gambler should take her $1 profit and quit. If the gambler loses this bet (which has probability $\frac{20}{38}$ of occurring), she should make additional $1 bets on red each of the next two spins of the roulette wheel and then quit. Let $X$ denote the gambler’s winnings when she quits.

There are five possible sequences of outcomes R, BRR, BRB, BBR, BBB, where R indicates red and B indicates black (or not red). The winnings associated with these events are +1, +1, -1, -1, -3 and they have probabilities $\frac{18}{38} \cdot \left(\frac{20}{38}\right) \cdot \left(\frac{18}{38}\right)$, $\left(\frac{20}{38}\right) \cdot \left(\frac{18}{38}\right)$, $\left(\frac{20}{38}\right)$, $\left(\frac{20}{38}\right)$, and $\left(\frac{20}{38}\right) \cdot \left(\frac{20}{38}\right) \cdot \left(\frac{20}{38}\right)$.

Thus

$$P(X > 0) = \frac{18}{38} + \left(\frac{20}{38}\right) \cdot \left(\frac{18}{38}\right) \cdot \left(\frac{18}{38}\right) \approx .5918.$$ 

No it is not a winning strategy because

$$E(X) = 1 \left(\frac{18}{38} + \left(\frac{20}{38}\right) \cdot \left(\frac{18}{38}\right) \cdot \left(\frac{18}{38}\right)\right)$$

$$+(-1) \left(\frac{20}{38} \cdot \left(\frac{18}{38}\right) \cdot \left(\frac{20}{38}\right) + \left(\frac{20}{38}\right) \cdot \left(\frac{20}{38}\right) \cdot \left(\frac{18}{38}\right)\right)$$

$$+(-3) \left(\frac{20}{38} \cdot \left(\frac{20}{38}\right) \cdot \left(\frac{20}{38}\right)\right)$$

$$\approx -.108$$

< 0.

21. $E(X)$ should be larger because

$$E(X) = \frac{40}{148} + \frac{33}{148} + \frac{25}{148} + \frac{50}{148} \approx 39.28$$
27. Let $C$ be the cost of the policy. If $E$ occurs then the company will lose $C - A$ dollars. This happens with probability $p$. If $E$ doesn’t occur then the company will gain $C$ dollars. This happens with probability $p$. Thus setting the expected profit equal to $.1A$ we get

\[ .1A = p(C - A) + (1 - p)C \]

which implies $C = A(1 + p)$.

30. (a) No. I very likely would lose all my money.
(b) Probably not. In theory I could make money if I played long enough, but it would take me so long that I might be dead before I was ahead.

31. If the meteorologist believes that the probability is $p^*$ then she believes her expected score is

\[ S(p^*) = (1-(1-p)^2)p^*+(1-p^2)(1-p^*) = (2p-p^2)p^*+(1-p^*)1-p^2(1-p^*). \]

We want to maximize this over all values of $p$

\[ S'(p) = (2 - 2p)p^* - 2p(1 - p^*) = 2p^* - 2p. \]

Setting this equal to 0 and solving for $p$ we get that $p = p^*$. Thus the meteorologist has an incentive to predict honestly.

32. For each group of 10 people they will either need 1 test (if everyone is healthy) or 11 if at least one person has the disease. Since the probability a person is healthy is $.9$ and each person is healthy independently of the others, the probability that everyone is healthy is $(.9)^{10}$ and the probability that at least one person is sick is $1 - (.9)^{10}$. Thus the expected number of tests is

\[ 11(1 - (.9)^{10}) + 1(.9)^{10}. \]

33. Let $D$ be the demand for papers. This random variable has a Bin$(10, 1/3)$ distribution. Let $P$ be the number of papers the boy purchases. Let $S_P$ be the number of papers sold if the paperboy bought $P$ papers. Then $S_P = \min(D, P)$. The paperboy’s expected profit is

\[ E(.15S_P - .1P). \]
Thus
\[ P(S_P = i) = \binom{10}{i} \left( \frac{2}{3} \right)^{10-i} \left( \frac{1}{3} \right)^i \]
for \(i < p\) and
\[ P(S_P = P) = \sum_{i \geq P} \binom{10}{i} \left( \frac{2}{3} \right)^{10-i} \left( \frac{1}{3} \right)^i. \]

We see the expected value is largest when \(P = 3\). See the excel spreadsheet for details.

35. The probability that the two balls are of the same color is
\[ \frac{(10)(4)}{(10)(9)} = \frac{4}{9}. \]
Thus the expected winnings are
\[ E(W) = \frac{4}{9}(1.1) + \frac{5}{9}(-1) = \frac{4.4 - 5}{9} = \frac{1}{15}. \]
Then
\[ E(W^2) = \frac{4}{9}(1.1)^2 + \frac{5}{9}(-1)^2 = \frac{4.84 + 5}{9} = \frac{9.84}{9} \]
and
\[ \text{Var}(W) = E(W^2) - E(W)^2 = \frac{9.84}{9} - \left( -\frac{1}{15} \right)^2 \approx 1.089. \]

37. (a)
\[ \text{Var}(X) = E(X^2) - E(X) \]
\[ = \frac{40}{148} (40)^2 + \frac{33}{148} (33)^2 + \frac{25}{148} (25)^2 + \frac{50}{148} (50)^2 \]
\[ - \left( \frac{40}{148} \frac{40}{148} + \frac{33}{148} \frac{33}{148} + \frac{25}{148} \frac{25}{148} + \frac{50}{148} \frac{50}{148} \right)^2 \]
\[ \approx 82.2. \]
(b)
\[ \text{Var}(Y) = E(Y^2) - E(Y) = \frac{1}{4} ((40)^2 + (33)^2 + (25)^2 + (50)^2) - (37)^2 = 84.5. \]
38. (a) \[ E((2 + X)^2) = E(4 + 4X + X^2) = 4 + 4E(X) + E(X^2) = 4 + 4(1) + \text{Var}(X) + E(X)^2 = 8 + 5 + 1^2 = 14. \]
(b) \[ \text{Var}(4 + 3X) = \text{Var}(3X) = 9\text{Var}(X) = 9(5) = 45. \]