Harmonic Analysis and Waves

Titles and Abstracts

Matthew Blair (U. New Mexico)

Title: Gaussian beam approximations on Riemannian manifolds and applications

Abstract: We consider high-frequency Gaussian beam approximations to the wave equation on a Riemannian manifold. These are approximate solutions whose phase space profile is highly concentrated along a geodesic on scales which saturate the uncertainty principle. In particular, we are interested in constructing approximations that are well-behaved under coordinate transformations and seek to analyze their dynamics over large, frequency-dependent time intervals. We then show how such constructions can be applied towards the analysis of eigenfunctions of the Laplacian on a compact Riemannian manifold. Of particular interest here are estimates on the microlocal Kakeya-Nikodym norms introduced in previous works with C. Sogge, which have implications for L^p bounds on eigenfunctions.

Tanya Christiansen (U. Colombia, Missouri)

Title: The semiclassical structure of the scattering matrix for a manifold with infinite cylindrical end

Abstract: We study the microlocal properties of the scattering matrix associated to the semiclassical Schrödinger operator $P = h^2\Delta_X + V$ on a Riemannian manifold with an infinite cylindrical end. Let Y denote the cross section of the end, which is not necessarily connected. We show that under suitable hypotheses, microlocally the scattering matrix is a Fourier integral operator associated to the graph of the scattering map $\kappa : \mathcal{D}_\kappa \to T^*Y$, with $\mathcal{D}_\kappa \subset T^*Y$. The scattering map κ and its domain $\mathcal{D}_\kappa$ are determined by the Hamilton flow of the principal symbol of P. As an application we prove that, under additional hypotheses on the scattering map, the eigenvalues of the associated unitary scattering matrix are equidistributed on the unit circle.

This talk is based on joint work with A. Uribe.

Alexis Drouot (UW)

Title: Semiclassical Dirac operators for topological insulators.

Abstract: We study a semiclassical Dirac equation on the plane that emerges in the study of topological insulators. When the crossing set (the set where the eigenvalues are repeated) is a one-dimensional submanifold of phase-space, we show that wavepackets initially localized at the crossing set split in two parts. The first one travels at speed and direction independent of the initial profile. The second one immediately collapses. This is consistent with the bulk-edge correspondence, which predicts that transport along interfaces of topological insulators is always asymmetric.

Mihaela Ifrim (U. Wisconsin, Madison)

Title: The time-like minimal surface equation in Minkowski space: low regularity solutions

Abstract: It has long been conjectured that for nonlinear wave equations which satisfy a nonlinear form of the null condition, the low regularity well-posedness theory can be significantly improved compared to the sharp results of Smith-Tataru for the generic case. The aim of this article is to prove the first result in this direction, namely for the time like minimal surface equation in the Minkowski space-time. Further, our improvement is substantial, namely by $3/8$ derivatives in two space dimensions and by $1/4$ derivatives in higher dimensions. This work is joint with Albert Ai and Daniel Tataru.

Jeffrey Galkowski (U. College London)

Title: The Pollution effect in Helmholtz boundary element methods

Abstract: In this talk we discuss boundary integral methods used to numerically solve sound-soft (Dirichlet) and sound-hard (Neumann) obstacle scattering problems in dimension d. We will review a few of these methods and consider one of the central questions about their accuracy: As the frequency, k, tends to infinity, how does dimension, N, of the approximation space need to increase to maintain accuracy of the solution?

For unstructured spaces, the natural minimal threshold for accuracy is $N \sim k^{d-1}$ and a method is said to suffer from the pollution effect if $N \gg k^{d-1}$ is required to maintain accuracy as $k \to \infty$. In this talk, we show that methods based on Fourier type bases do not (or nearly do not) suffer from the pollution effect even when the underlying domain is trapping. However, for methods based on piecewise polynomials, we show that there is pollution unless the domain is non-trapping and give sharp lower bounds on N required to maintain accuracy.

Katrina Morgan (Northwestern)

Title: Wave propagation on rotating cosmic string backgrounds

Abstract: A rotating cosmic string spacetime has a singularity along a timelike curve corresponding to a one-dimensional source of angular momentum. Such spacetimes are not globally hyperbolic: they admit closed timelike curves near the so-called "string". This presents challenges to studying the existence of solutions to the wave equation via conventional energy methods. In this work, we show that forward solutions to the wave equation (in an appropriate microlocal sense) do exist. Our techniques involve proving a statement on propagation of singularities and using the resulting estimates to show existence of solutions. This is joint work with Jared Wunsch.

Jason Metcalfe (UNC, Chapel Hill)

Title: Almost global existence for 3-d systems of nonlinear wave equations

Abstract: In this joint work with Taylor Rhoads, we combine a space-time Klainerman-Sobolev estimate with an adaptation of Dafermos and Rodnianski's r^p-weighted local energy estimate to establish almost global existence for quasilinear wave equations with small initial data. The nonlinearity $Q(u, \partial u, \partial^2 u)$, which vanishes to second order, is permitted to depend on the solution, not just its derivatives. And we assume $(\partial_u^2 Q)(0,0,0) = 0$. The almost global result that is obtained is an analog of that proved by Lindblad for scalar equations.

Andreas Seeger (U. Wisconsin, Madison)

Title: L^p-improving properties of spherical maximal operators with restricted dilation sets.

Abstract: We consider local versions of spherical maximal operators with a supremum taken over a given dilation set. The type set of such an operator is the set of all $(1/p, 1/q)$ for which we have $L^p \to L^q$ boundedness. Type sets may depend on various notions of fractal dimensions of the dilation sets, such as the Minkowski and Assouad dimensions, and the Assouad spectrum of dimensions. The talk will focus on a simple characterization of the closed convex sets which can occur as closure of the type set for such maximal operators. This is joint work with Joris Roos.

Christoffer Sogge (Johns Hopkins)

Title: Product manifolds with improved spectral cluster and Weyl remainder estimates

Abstract: We describe joint work with Xiaoqi Huang and Michael Taylor. We explore the problem of when one can obtain improved spectral and Weyl remainder estimates, especially on product manifolds. In particular, for products of spheres of length 5 or more we obtain optimal L^q estimates for eigenfunctions for q sufficiently large in a result that generalizes a classical result of Walfisz for the torus. This result, although for a special class of manifolds and involving large exponents, is a natural analog of the Stein-Tomas restriction/extension theorem for Euclidean space. It and other of our results are also motivated by recent work of Iosevich and Wyman.

Mikko Salo (U. Jyväskylä)

Title: The fixed angle inverse scattering problem

Abstract: We consider the fixed angle inverse scattering problem for potentials in Euclidean space. The main result shows that a compactly supported potential is uniquely determined by its scattering amplitude for two opposite fixed angles. We also show that potentials having a generalized reflection symmetry are determined by their fixed angle scattering data. The results are based on wave equation methods: one reduces the inverse scattering problem into a unique continuation type problem for the wave equation, and then uses suitable Carleman estimates in the spirit of the Bukhgeim-Klibanov method. We will also discuss extensions to more general settings.

Daniel Tataru (UC Berkeley)

Title: Global solutions for 1D cubic dispersive flows

Abstract: The question of obtaining scattering, global in time solutions for such problems has attracted a lot of attention in recent years, and many global well-posedness results have been proved for a number of models under the assumption that the initial data is both small and localized. However, except for the completely integrable case, no such results have been known for small but non-localized initial data. In this work we introduce a new, nonperturbative method, to prove global well-posedness and scattering for L2 initial data which is small but non-localized. Our main structural assumption is that our nonlinearity is defocusing. In particular we do not assume that our problem has any exact conservation laws. This is joint with Mihaela Ifrim.

Maciej Zworski (UC Berkeley)

Title: Internal waves in 2D aquaria and homeomorphisms of the circle

Abstract: The connections between the formation of internal waves in fluids, spectral theory, and homeomorphisms of the circle were investigated by oceanographers in the 90s and resulted in novel experimental observations (Leo Maas et al, 1997). The specific homeomorphism is given by a "chess billiard" and has been considered by many authors (Fritz John 1941, Vladimir Arnold 1957, Jim Ralston 1973...). The relation between the nonlinear dynamics of this homeomorphism and linearized internal waves provides a striking example of classical/quantum correspondence (in a classical and surprising setting of fluids!). I will illustrate the results with numerical and experimental examples and explain how classical concepts such as rotation numbers of homeomorphisms (introduced by Henri Poincare) are related to solutions of the Poincare evolution problem (so named by Elie Cartan). The talk is based on joint work with Semyon Dyatlov and Jian Wang.