# Review for Math 120 based on Math 120 Assessment

Here you will find review material, examples and practice problems corresponding to each problem on the Math 120 Assessment. If you were unable to solve any of those problems, look here for suggestions for how to approach such problems.

1. Assessment problem: *Fifteen years ago, a person was half as old as they will be five years from now. How old are they now?*

   Problems of this sort require you to identify unknowns, translate an English sentence into an equation, and then solve the equation.

   **Example:** In 20 years a person will be twice as old as they were five years ago . How old are they now?

   For this problem the unknown is the age of the person, let's call it $x$; the problem tells us that

   $x + 20$ (age of the person 20 years from now)= $2(x - 5)$ (age of the person 5 years ago)

   We can then solve for $x$:

   $$\begin{aligned} x + 20 &= 2x - 10 \\ x - 2x &= -10 - 20 \\ -x &= -30 \\ x &= 30 \end{aligned}$$

   Thus, the person is 30 years old now.

   **Practice problems**

   (a) The number of hours that were left in the day was one-third of the number of hours already passed. How many hours were left in the day? (Ans.: 6.)

   (b) When 6 is added to four times a number the result is 50. Find the number. (Ans.: 11.)

   (c) The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket. (Ans.: senior citizen ticket: $8, child ticket: $14.)

   Note that this example is a little bit different from the first two and you can check problem 5 for a review of the algebra needed to solve a system of two linear equations in two variables.

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2. Assessment problem: *Starting today, the height of a certain tree will increase by $3\%$ each year for the next six years, and then decrease by $5\%$ over the following year. Compared to today, how tall will the tree be seven years from now?*

   When an original amount $A_0$ increases or decreases by a constant percentage $r$ (in decimal form) each unit time, the amount at time $t$ is given by an exponential function:

   $$f(t) = A_0(1 + r)^t \quad \text{for an increase}$$

   and

   $$g(t) = A_0(1 - r)^t \quad \text{for a decrease.}$$

   **Example:** A country debt decreased by $2\%$ each year for the past 10 years, but it is expected to increase $3\%$ each year for the next two years. How does the country debt two years from now compare to the debt 10 years ago ?

   Let $A_0$ be the country debt 10 years ago, then this year the debt is $A_0(1 - 0.02)^{10}$ and two years from now the debt will be $A_1 = A_0(1 - 0.02)^{10}(1 + 0.03)^2 = .8668A_0$. Therefore, two years from now the debt will be 13.32% (1-.8668=.1332) less than 10 years ago.

   **Practice problems**

(a) In 1985 there 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were there in Centerville in 1994 ? (Ans.: 43872.)

(b) Assume that a bacteria population increases at a constant rate $r$ each hour. Initially there are 100 bacteria and after 5 hours the number of bacteria is 2200. Find r. (Ans.: .8556 or $85.56\%$ )

(c) You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by $12\%$. How long until you have 10mg of caffeine ? (Ans.: 19.4 hours)

Note: you need to be able to solve an exponential equation to solve this problem.

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3. Assessment problem: *Find the equation of the line which passes through the points* $(1,3)$ *and* $(-2,-5)$.

To find the equation of a line through two given points $(x_1,y_1)$ and $(x_2,y_2)$, begin by finding the **slope** $m$ of the line via the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

This works perfectly unless $x_1 = x_2$ in which case the line is vertical; in that case, the equation of the line is $x = x_1$

Then use the **point-slope** formula for a line:

$$y = m(x - x_1) + y_1.$$

We can use this form any time we know the slope of a line and a point that it passes through.

**Example:** Find the equation of the line $L$, that passes through $(5,4)$ and $(10,1)$.

Since $L$ passes through $(5,4)$ and $(10,1)$, it has slope

$$\frac{1-4}{10-5} = -\frac{3}{5}.$$

Hence, it has equation $y = -\frac{3}{5}(x-5)+4$.

**Practice problems**

(a) Find the equation of the line through $(1,2)$ and $(0,1)$. (Ans.: $y = 1 + x$ )

(b) Find the equation of the line through $(1,2)$ and $(1,1)$. (Ans.: $x = 1$. )

(c) Find the equation of the line with $x$ intercept 3, through $(1,2)$ , (Ans.: $y = -x + 3$ )

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4. Assessment problem: *Find the equation of the line which passes through the point* $(3,5)$ *and is perpendicular to the line with equation* $2x - 3y = 5$.

You can use the **point-slope** formula for a line of slope $m$ going through the point $(x_1,y_1)$:

$$y = m(x - x_1) + y_1.$$

We can use this form any time we know the slope of a line and a point that it passes through. You also need to remember that if a line has slope $m_1$ and another line has slope $m_2$, then the lines are perpendicular when $m_1 m_2 = -1$, or, equivalently,

$$m_2 = \frac{-1}{m_1}.$$

You do not need it for this problem, but you should also remember that the lines are parallel if $m_1 = m_2$.

**Example:** Find the equation of the line perpendicular to the line $y = 5x + 8$ and passing through the point $(0,6)$.

Line $B$ is perpendicular to $y = 5x + 8$ which has slope 5. Hence, line $B$ has slope $-\frac{1}{5}$ and equation

$$y = -\frac{1}{5}(x-0) + 6 = -\frac{1}{5}x + 6.$$

(a) Let line $A$ be the line $y = 5(x-1)+3$. Find the equation of the line that is perpendicular to line $A$ and passes through line $A$'s $y$-intercept. (Ans.: $y = -\frac{1}{5}x - 2$. )

(b) Let line $A$ be the line $y = 5(x-1)+3$. Find the equation of the line that is parallel to line $A$ and passes through line $A$'s $x$-intercept (Ans.: $y = 5(x+\frac{2}{5})$. )

(c) Let line $A$ be the line through $(7,0)$ and $(1,5)$. Let line $B$ be the line perpendicular to line $A$ which passes through the point $(10,0)$. Find the point of intersection of lines $A$ and $B$. (Ans.: $(535/61, -90/61)$)

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5. Assessment problem: *Find the solution to the following pair of equations:*

$$3x + 4y = 8$$
$$2x - 5y = 10$$

To solve a linear system of two equations in two variables $x$ and $y$ you can :

(a) Solve for $x$ in the first equation. You will get $x =$ some expression containing $y$.

(b) Plug in the expression in $y$ for $x$ in the second equation. You will get a linear equation in $y$ (no more $x$). You can solve for $y$. Let's call your solution $y_1$.

(c) Plug $y_1$ back in the expression you found for $x$ in step (a) . This gives you a value $x_1$.

(d) The solution to your system is $x = x_1$, $y = y_1$.

**Example:**

Solve the following system of equations. $\begin{cases} 2x + 3y = 1 \\ x - \ \ y = 2 \end{cases}$

(a) Solve for $x$ in the first equation: $x = \frac{1}{2} - \frac{3}{2}y$.

(b) Substitute $\frac{1}{2} - \frac{3}{2}y$ for $x$ in the second equation to get $(\frac{1}{2} - \frac{3}{2}y) - y = 2$ and solve for $y$: $y = -\frac{3}{5}$.

(c) Plug $-\frac{3}{5}$ in for $y$ in the expression you found for $x$ in step (a): $x = \frac{1}{2} - \frac{3}{2}(-\frac{3}{5}) = \frac{7}{5}$.

(d) The solution to your system is $x = \frac{7}{5}$, $y = -\frac{3}{5}$.

**Practice problems:**

(a) Solve $\begin{cases} 2x + 5y = 1 \\ 3x - 2y = 2 \end{cases}$ (Ans.: $x = \frac{12}{19}, y = -\frac{1}{19}$)

(b) Solve $\begin{cases} x + 7y = 0 \\ x - \ \ y = 3 \end{cases}$ (Ans.: $x = \frac{21}{8}, y = -\frac{3}{8}$)

(c) Solve $\begin{cases} 2x - \ \ y = 1 \\ 3x + 4y = 5 \end{cases}$ (Ans.: $x = \frac{9}{11}, y = \frac{7}{11}$)

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6. Assessment problem: *Find all solutions to the equation* $3x^2 + 5x = 2$.

To find the solutions to a quadratic equation $ax^2 + bx + c = 0$ we can use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This will give two distinct solutions if $b^2 - 4ac > 0$, one solution if $b^2 - 4ac = 0$ and no solutions if $b^2 - 4ac < 0$.

**Example:** Find all solutions to the equation $3x^2 + 5x = 1$.

First we rewrite the equation as $3x^2 + 5x - 1 = 0$, so $a = 3, b = 5, c = -1$. The quadratic furmula gives 2 solutions $x = \frac{-5+\sqrt{5^2-4(3)(-1)}}{6} = \frac{-5+\sqrt{37}}{6}$ and $x = \frac{-5-\sqrt{5^2-4(3)(-1)}}{6} = \frac{-5-\sqrt{37}}{6}$

**Practice problems**: Solve the following quadratic equations:

(a) $5x^2 - 5x - 10 = 0$. (Ans.: $x = -1, 2$.)

(b) $12x^2 - 12x + 3 = 0$. (Ans.: $x = \frac{1}{2}$)

(c) $3x^2 + x + 5 = 0$ (Ans.: No solutions.)

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7. Assessment problem: *Find all solutions to the equation* $(3x+1)(x+2) - x(x-3) = x^2 + 5$.

   For this kind of problems you should recognize that if you multiply $(3x+1)(x+2)$ and $x(x-3)$ you obtain monomials in $x$ and $x^2$ , so your goal is to multiply out and simplify and rewrite the equation in the form $ax^2 + bx + c = 0$. You can then use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve the equation. This will give two distinct solutions if $b^2 - 4ac > 0$, one solution if $b^2 - 4ac = 0$ and no solutions if $b^2 - 4ac < 0$.

   **Example:** Find all solutions to the equation $(4x+1)(x+2) - 4x(x+1) = -3x^2 + 1$.

   First multiply out:

   $$4x^2 + 8x + x + 2 - 4x^2 - 4x = -3x^2 + 1,$$

   then simplify to $3x^2 + 5x - 1 = 0$, so $a = 3, b = 5, c = -1$. The quadratic formula gives two solutions $x = \dfrac{-5 + \sqrt{5^2 - 4(3)(-1)}}{6} = \dfrac{-5 + \sqrt{37}}{6}$ and $x = \dfrac{-5 - \sqrt{5^2 - 4(3)(-1)}}{6} = \dfrac{-5 - \sqrt{37}}{6}$.

   **Practice problems**

   Solve the following equations:

   (a) $x(2x-1) = (x+2)(x-3)$. (Ans.: No solutions).

   (b) $(3x+1)x - (x-3) = x(x-7)$. (Ans.: $x = -\frac{1}{2}, -3$)

   (c) $3(x-1)(x-2) = (2x-7)x + 5$ (Ans.: $x = 1$)

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8. Assessment problem: *Find all solutions to the equation* $3 = \dfrac{1}{x} + \dfrac{2}{x-5}$.

   In this example, the variable $x$ appears in the denominator of a fraction. To solve this kind of equations you should:

   - Find the lowest common denominator (LCD) of all fractions that appear in the equation.
   - Multiply both sides of the equation by the LCD (the product of all denominators will work).
   - Simplify (no more fractions) and solve the equation.
   - Discard any solution you found that makes any denominator of the original equation equal to 0.

   **Example:** Solve $2 = \dfrac{3}{x-1} + \dfrac{2}{x-5}$.

   - Find the lowest common denominator (LCD) : $(x-1)(x-5)$
   - Multiply both sides of the equation by the LCD : $2(x-1)(x-5) = (\dfrac{3}{x-1} + \dfrac{2}{x-5})(x-1)(x-5)$
   - Simplify : $2(x^2 - 5x - x + 5) = 3(x-5) + 2(x-1)$ becomes $2x^2 - 17x + 27 = 0$.
   - Solve the equation: you can use the quadratic formula to solve $2x^2 - 17x + 27 = 0$. The solutions are $x_1 = \frac{17+\sqrt{73}}{4}$ and $x_2 = \frac{17-\sqrt{73}}{4}$.
   - Discard any solution you found that makes any denominator of the original equation equal to 0: since $x_1 - 1 \neq 0, x_1 - 5 \neq 0, x_2 - 1 \neq 0, x_2 - 5 \neq 0$ we keep both solutions. Therefore, our original equation has two solutions: $x_1 = \frac{17+\sqrt{73}}{4}$ and $x_2 = \frac{17-\sqrt{73}}{4}$.

   **Practice problems**

   Solve the following equations:

   (a) $x = \dfrac{3}{x-1} + \dfrac{x}{3}$. (Ans.: $\frac{1+\sqrt{19}}{2}, \frac{1-\sqrt{19}}{2}$ )

(b) $\dfrac{1}{x} = \dfrac{3}{x-1} + \dfrac{1}{2x}$. (Ans.: $-\frac{1}{5}$)

(c) $\dfrac{2}{x-1} = \dfrac{x+1}{x-1} + \dfrac{3}{x-2}$. (Ans.: $-1$)

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9. Assessment problem: *Find all solutions to the equation* $|3x+4| = 22$.

We need to remember the multipart definition of $|x|$:

$$|x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$$

Therefore, an equation of the form $|ax+b| = c$ splits into two different equations:

- $ax + b = c$ if $ax + b \geq 0$ , that is we only keep solutions $x$ such that $x \geq -\frac{b}{a}$
- $-(ax+b) = c$ if $ax + b < 0$ , that is we only keep solutions $x$ such that $x < -\frac{b}{a}$

**Example:** Find all solutions to the equation $|2x+1| = 3$.

We need to solve:

- $2x + 1 = 3$ (valid if $2x + 1 \geq 0$ ), and only keep solutions $x$ such that $x \geq -\frac{1}{2}$ ; $x = 1$ is our solution.
- $-(2x+1) = 3$ (valid if $2x+1 < 0$) , and only keep solutions $x$ such that $x < -\frac{1}{2}$; $x = -2$ is our solution.

Therefore, the equation has two solutions, $1$ and $-2$.

**Practice problems**

Solve the following equations:

(a) $|3x-5| = 2$ (Ans.: $1, \frac{7}{3}$)

(b) $|3x-5| = 2x$ (Ans.: 1,5)

(c) $|3x-5| = -4|x| + 5$ (Ans.: 0)

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10. Assessment problem: *Solve the inequality:* $14 - 3x < 6 + x$

To solve a linear inequality, you can perform the same steps that you would use to solve a linear equation.

You can add or subtract the same quantity from both sides of the inequality.

You can multiply/divide both sides of the inequality by a positive quantity.

You can multiply/divide both sides of the inequality by a negative quantity, but in this case you need to reverse the direction of the $>$ sign.

Your goal is to end with an inequality of the form $x <$ some number; or $x >$ some number.

**Example:** Solve the inequality: $1 - 2x < 5 + x$.

| | |
|---|---|
| $1 - 2x < 5 + x$ | subtract 1 from both sides |
| $-2x < 4 + x$ | subtract $x$ from both sides |
| $-3x < 4$ | divide both sides by -3 ($<$ turns into $>$) |
| $x > -\dfrac{4}{3}$ | the final result |

**Practice problems**

Solve the following inequalities:

(a) $2x + 1 > x - 5$ (Ans.: $x > -6$)

(b) $-2x + 1 > x - 5$ (Ans.: $x < 2$)

(c) $2x + 1 > 3x - 5$ (Ans.: $x < 6$)

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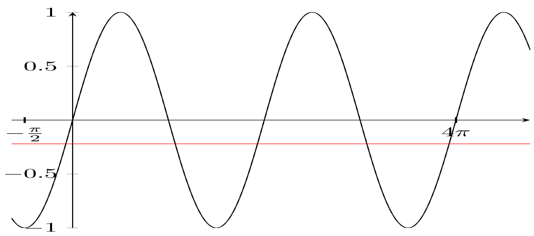
11. Assessment problem: *How many values of $x$ are there such that $\sin x = -0.22$ and $-\frac{\pi}{2} \le x \le 4\pi$?*

You should begin a problem like this by sketching the graph of $\sin x$ for at least the range of $x$ values indicated. Often a good understanding the graph will be all that you need to solve such a problem.




If we are asked to *find* the solutions to the given equation as opposed to just count them, then inverse sine often must be used.

It is good to remember that if $-1 \le a \le 1$, then the equation $\sin x = a$ has infinitely many solutions. We can find *one* of them via the inverse sine function. Let $P = sin^{-1}a$. Then $\sin P = a$ so $P$ is a solution to the equation $\sin x = a$. $P$ is often called the principal solution.

An important feature of the graph $y = \sin x$ is that it is symmetric about the vertical line $x = \dfrac{\pi}{2}$. As a result, if $P$ is a solution to $\sin x = a$, then we will also have a **symmetric** solution, $S$, given by

$$S = \frac{\pi}{2} + \left(\frac{\pi}{2} - P\right).$$

That is, there will be a solution as far to the right of $\frac{\pi}{2}$ as $P$ is the left of $\frac{\pi}{2}$.

Due to the nature of $y = \sin x$, all other solutions are shifts of $P$ and $S$ by multiples of $2\pi$, the period of sine.

**Example:** Find all solutions to $\sin x = 0.8$ with $-5 \le x \le 5$.

We begin by finding the principal solution: $P = \sin^{-1} 0.8 = 0.9272952180$. The symmetry solution is then $S = \frac{\pi}{2} + \frac{\pi}{2} - P = 2.2142974355$. If we add $2\pi$ to $P$, we get a value greater than 5. If we subtract $2\pi$ from $P$ we get a value less then $-5$. If we add $2\pi$ to $S$, we get a value greater than 5. If we subtract $2\pi$ from $S$, we get $-4.06888787159$, which is in our desired interval. Subtracting $2\pi$ again, though, would give a value less than $-5$.

Thus, our solutions are $0.9272952180, 2.2142974355$ and $-4.06888787159$.

**Practice problems:**

(a) How many solutions are there to $\sin x = 0.1$ with $-\pi \le x \le 2\pi$? (Ans.: 2.)

(b) How many solutions are there to $\sin x = -0.8$ with $-3\pi \le x \le 2\pi$? (Ans.: 6.)

(c) How many solutions are there to $\sin x = 0.3$ with $5 \le x \le 10$.? (Ans.: Use your knowledge of the graph of $\sin x$ together with the facts that $\pi < 5 < 2\pi$ and $3\pi < 10 < 4\pi$ to conclude that there is exactly one solution.)

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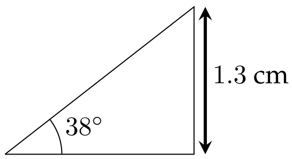
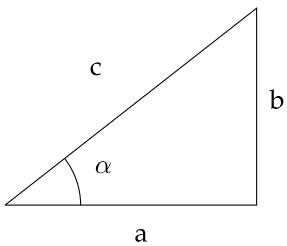
12. Assessment problem: *How long is the hypotenuse of the right triangle shown below?*




To solve this kind of problem, you need to remember the trigonometric ratios for sine, cosine and tangent. We can define them using this picture:




$$\sin\alpha = \frac{b}{c}$$
$$\cos\alpha = \frac{a}{c}$$
$$\tan\alpha = \frac{b}{a}$$

Make sure to check that your calculator is set in the correct mode, radians or degrees, when you compute with angles.

**Example:** How long is the hypotenuse of the triangle above if $\alpha = 0.6$ rad and $a = 2$ cm

In this case $\cos 0.6 = \frac{2}{c}$ and hence $c = \frac{2}{\cos 0.6} \approx 2.4233$ cm.

**Practice problems**

(a) Suppose $\alpha = 0.5$ rad in the triangle above and $c = 3$ cm, find $b$. (Ans.: $1.4383$ cm)

(b) Suppose $\alpha = 24°$ in the triangle above and $b = 3$ cm, find $a$. (Ans.: $6.7381$ cm)

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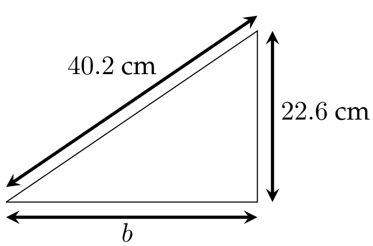
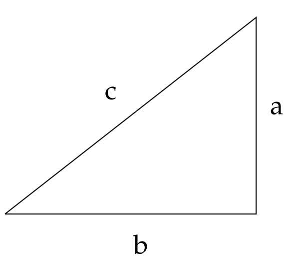
13. Assessment problem: *Find the length b in the figure below*




We need to remember the Pythagorean theorem: for a right triangle like the one below, $c^2 = a^2 + b^2$.




**Example:** Find the length of a in the triangle above, if b=2 and c=5.

Using the Pythagorean theorem $a = \sqrt{5^2 - 2^2} = \sqrt{21}$

**Practice problems**

(a) Suppose the hypotenuse of a right triangle is $10$ meters long and one leg is $8$ meters long. How long is the other leg? (Ans.: $6$ meters.)

(b) Suppose the legs of a right triangle have length $4$ meters and 7 meters. How long is the hypotenuse? (Ans.: $\sqrt{65}$ meters.)

(c) Suppose one leg of a right triangle is $1$ meter long and the hypotenuse is twice as long as the other leg. How long is the hypotenuse? (Ans.: $\frac{2}{\sqrt{3}}$ meters.)

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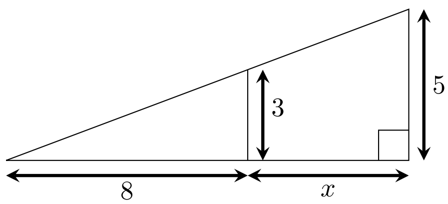
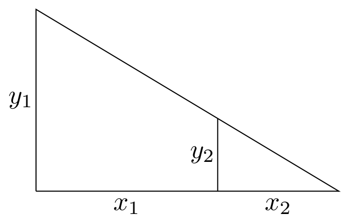
14. Assessment problem: *Find the unknown length $x$ in the figure below.*




Problems of this sort require you to apply the concept of **similar triangles**.

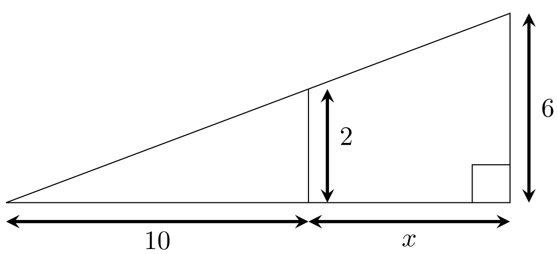
Consider the right triangular figure below.




Similar triangles tells us that $\dfrac{y_1}{x_1 + x_2} = \dfrac{y_2}{x_2}$.

If we know all but one of the quantities $x_1, x_2, y_1$, and $y_2$, we can solve the above equation for that unknown value.

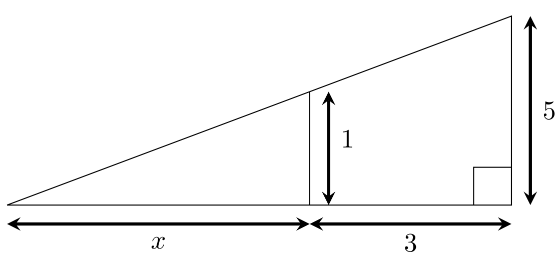
**Example:** Find the unknown length $x$ in the triangle below:




Using similar triangles , we can write the proportion $\frac{10+x}{6} = \frac{10}{2}$ and then solve for $x$ $10 + x = 30$, $x = 20$

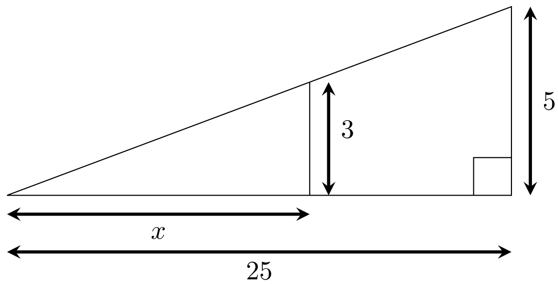
**Practice problems**

(a) Find the unknown length $x$ in the triangle below




(Ans.: $\frac{3}{4}$)

(b) Find the unknown length $x$ in the triangle below




(Ans.:15 )

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15. Assessment problem: *Suppose* $f(x) = 4x + 5$. *Find all solutions to the equation* $f(x + 3) = 2f(2x)$

    This problem requires you to evaluate the composition of two functions and then to solve a linear equation. Given two functions $f(x)$ and $g(x)$, in order to evaluate $f(g(x))$ you may want first to rewrite the formula for $f$ replacing $x$ with $z$; now you have an expression $f(z)$ containing the variable $z$ and you need to replace any $z$ in the expression with the $g(x)$.

    **Example:** Suppose $f(x) = 2x - 3$. Find all solutions to the equation $f(x - 5) = 3f(2x)$.

    First we need to evaluate $f(x - 5)$ and $f(2x)$, then we will need to solve an equation.

    Let's write $f(z) = 2z - 3$.

    To calculate $f(x - 5)$, we need to replace all $z$s with $x - 5$: we get $f(x - 5) = 2(x - 5) - 3 = 2x - 13$.

    To calculate $f(2x)$, we need to replace all $z$ with $2x$: we get $f(2z) = 2(2x) - 3 = 4x - 3$.

    Thus the equation $f(x - 5) = 3f(2x)$ can be written $2x - 13 = 3(4x - 3)$.

    Solving this equation, we find $x = -\frac{2}{5}$, the one solution to $f(x - 5) = 3f(2x)$.

    **Practice Problems**

    (a) Let $f(x) = 3x + 2$. Find all solutions of $2 - f(3x) = f(5 + 2x)$ (Ans.: $x = -\frac{17}{15}$)

    (b) Let $f(x) = 3x - 2$, For which values of $a$ is $x = 1$ a solution of the equation $2 - f(3x) = f(a + 2x)$ ? (Ans.: $a = -3$)

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16. Assessment problem: *Suppose* $g(x) = x^2 - x$. *Find all solutions to the equation* $g(2x) = 3g(x + 1)$.

    This problem requires you to evaluate the composition of two functions and then to solve a quadratic equation. Given two functions $f(x)$ and $g(x)$, in order to evaluate $f(g(x))$ you may want first to rewrite the formula for $f$ replacing $x$ with $z$; now you have an expression $f(z)$ containing the variable $z$, and you need to replace any $z$ in the expression with $g(x)$

    **Example:** Suppose $f(x) = x^2 + x$ Find all solutions to the equation $f(x - 5) = 3f(2x)$

    First we need to evaluate $f(x - 5)$ and $f(2x)$, then we will need to solve an equation.

    Let 's write $f(z) = z^2 + z$.

    To calculate $f(x - 5)$ we need to replace all $z$ with $x - 5$:

    $$f(x - 5) = (x - 5)^2 + (x - 5) = x^2 - 9x + 20.$$

    To calculate $f(2x)$, we need to replace all $z$ with $2x$:

    $$f(2z) = (2x)^2 + (2x) = 4x^2 + 2x.$$

    Therefore, we need to solve the quadratic equation

    $$x^2 - 9x + 20 = 3(4x^2 + 2x)$$

which simplifies to

$$11x^2 + 15x - 20 = 0.$$

We can solve this equation using the quadratic formula and find $x = \frac{-15 \pm \sqrt{15^2 - 4 \cdot 11 \cdot (-20)}}{22}$ so the solutions are $x = \frac{-15+\sqrt{1105}}{22}$ and $x = \frac{-15-\sqrt{1105}}{22}$.

**Practice Problems**

(a) Let $f(x) = x^2 - 2x$. Find all solutions to $2 + f(3x) = f(5 + 2x)$ (Ans.: $x = \frac{9 \pm \sqrt{246}}{5}$)

(b) Let $f(x) = 2x^2 - 3x$. Find all solutions to $f(4x - 1) = f(x + 6)$. (Ans.: $x = -\frac{7}{10}, x = \frac{7}{3}$.)

(c) Let $f(x) = x^2 + 3x + 1$. Find all solutions to $f(2x) = 2f(x)$. (Ans.: $x = \pm\frac{1}{\sqrt{2}}$.)

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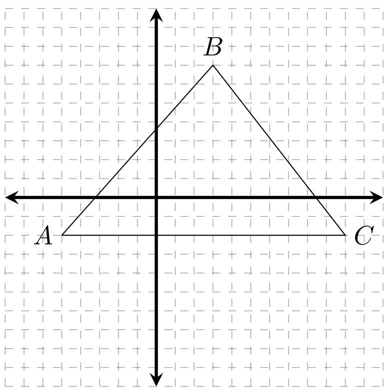
17. Assessment problem: *What is the area of the triangle in the $xy$-plane with vertices $(-5, -3)$, $(3, 7)$ and $(10, -3)$?*

For this problem it is useful to graph the vertices in the $xy$ plane first and then use the formula.
Area of triangle = $\frac{1}{2}$ base · height.

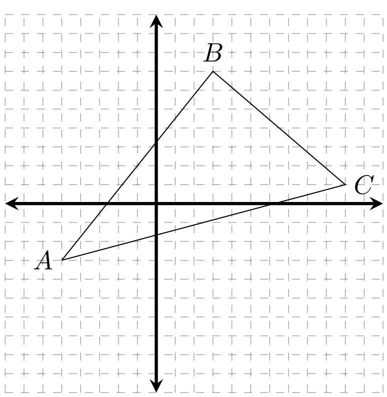
**Example:** What is the area of the triangle in the $xy$-plane with vertices $A = (-5, -2)$, $B = (3, 7)$ and $C = (10, -2)$?



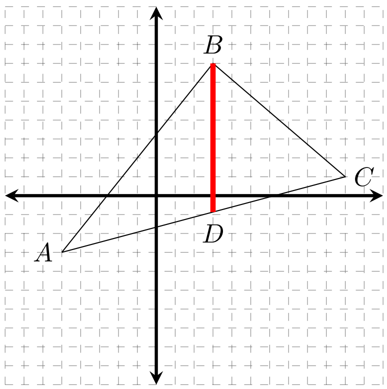

In this case we can see that the base $AC$ of the triangle is parallel to the $x$ axis and has length 10-(-5)=15 and the height is parallel to the $y$ axis and has length 7+2=9, therefore AREA= $\frac{1}{2} 15 \cdot 9 = \frac{135}{2}$

When the sides of the Triangle are not parallel to the axes the calculations are more complicated .

**Example:** What is the area of the triangle in the $xy$-plane with vertices $A = (-5, -3)$, $B = (3, 7)$ and $C = (10, 1)$?




To find the area, think of cutting the triangle in half with a vertical line through $B$.




Let $D$ be the point of intersection of this line and the line $AC$.

Then find the areas of triangle $ABD$ and triangle $BCD$ and add.

The hardest part is finding the length of $BD$. To do that, find the equation of line $AC$ and plug in the $x$-coordinate of $D$ to find the $y$-coordinate of $D$.

The line through $A$ and $C$ is $y = \frac{4}{15}(x - 10) + 1$, so $D$ has $y$-coordinate equal to $-\frac{13}{15}$.

Hence, the distance from $B$ to $D$ is $7 - \left(-\frac{13}{15}\right) = \frac{118}{15}$.

Then the area of triangle $ABD$ can be found using $BD$ as the base, and the height determined by point $A$ and the base $BD$. Likewise, the area of triangle $BCD$ can be found using $BD$ as the base, and the height determined by the point $C$ and the base $BD$.

Thus the area of triangle $ABD$ is $\frac{1}{2}\frac{118}{15}(3 - (-5)) = \frac{472}{15}$ and the area of triangle $BCD$ is $\frac{1}{2}\frac{118}{15}(10 - 3) = \frac{413}{15}$.

Adding the areas of triangle $ABD$ and triangle $BCD$ together gives us the area of the entire triangle, $59$.

**Practice problems**

(a) What is the area of the triangle in the $xy$-plane with vertices $A = (-5, -5)$, $B = (3, 1)$ and $C = (-1, -5)$?(Ans.: 18)

(b) What is the area of the triangle in the $xy$-plane with vertices $A = (-5, -5)$, $B = (3, 1)$ and $C = (4, -2)$? (Ans.: 15)

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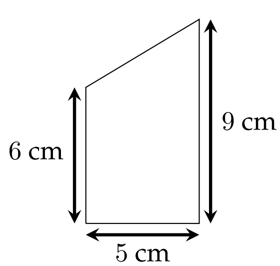
18. Assessment problem: *What is the area of the trapezoid shown below?*




There are just a few formulas for the areas of certain plane figures that you should be ready to use.

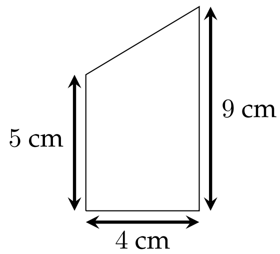
A rectangle with sides of length $a$ and $b$ has an area of $ab$.

A triangle with base length $b$ and height $h$ has an area of $\frac{1}{2}bh$.

A circle of radius $r$ has an area of $\pi r^2$.

A trapezoid with heights $h_1$ and $h_2$ and height $w$ has an area of $\frac{1}{2}(h_1 + h_2)w$.

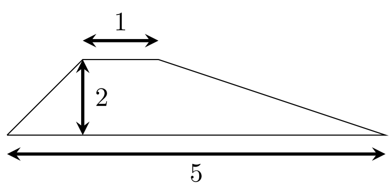
**Example:** What is the area of the trapezoid shown below ?




The two parallel sides have lengths $h_1 = 5$ and $h_2 = 9$. The two parallel sides are separated by a width $w = 4$. Thus, the area of the trapezoid if $\frac{1}{2}(5+9)(4) = 28\ \text{cm}^2$.

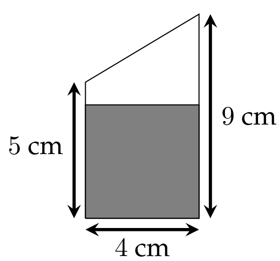
**Practice Problems**

(a) Find the area of the trapezoid below




(Ans.: 6)

(b) Find the area of the unshaded region, knowing that the shaded figure is a square.




(Ans.:12 )