

PROBLEM SET 3 (due on Friday, February 22nd)

FROM THE TEXT:

Section 6.4: Problems 5 b, c, f, (just the left cosets for part f), 6, 9.

Section 9.3: Problems 8, 9, 48, 50 .

ADDITIONAL PROBLEMS:

A: Let $G = Q_8$. Let $H = \langle -1 \rangle$. Let $K = \langle i \rangle$. Both H and K are subgroups of G . Find the left cosets of H in G . Find the right cosets of H in G . Find the left cosets of K in G . Find the right cosets of K in G .

B: Let $G = S_3$. Let $H = \langle (1\ 2) \rangle$. Find the left cosets of H in G . Find the right cosets of H in G .

C: Suppose that G is a group and that $c \in G$. Let $H = \{ h \in G \mid hc = ch \}$. Thus, H is the set of elements in G which commute with c .

(a) Prove that H is a subgroup of G .

(b) Suppose that $d \in G$ and that d is conjugate to c in G . Prove that the set

$$\{ a \in G \mid aca^{-1} = d \}$$

is a left coset of H in G .

D: Let $G = S_4$. Let $H = \{ \sigma \in G \mid \sigma(4) = 4 \}$.

(a) Prove that H is a subgroup of G and that $|H| = 6$.

(b) Suppose that $j \in \{1, 2, 3, 4\}$. Prove that the set $\{ \sigma \in G \mid \sigma(4) = j \}$ is a left coset of H in G .