A: This problem concerns the group $S_4$. Let $H = \{ f \mid f \in S_4, \ f(4) = 4 \}$. As shown in the previous homework assignment, $H$ is a subgroup of $S_4$ and has order 6. The same argument shows that, for any $j \in \{1, 2, 3, 4\}$, the set $H_j = \{ f \mid f \in S_4, \ f(j) = j \}$ is a subgroup of $S_4$ and has order 6. The subgroup $H$ defined before is just the special case where $j = 4$. (This subgroup $H_j$ is sometimes called the “stabilizer” of $j$ in $S_4$.) In this problem, I want you to prove that the subgroups $H_j$ are all conjugate to $H$. More precisely, suppose that $y \in G$ and that $y(4) = j$. Prove that

$$yHy^{-1} = H_j$$

Also, prove that the four subgroup $H_1, H_2, H_3$ and $H_4$ of $S_4$ are different. Determine $H_1 \cap H_4$. 

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