MATH 310 - Suggested exercises about equivalence relations and congruences.

From the text: Pages 271-273: 1, 7, 17

A: Find the last digit of $2^{11213} - 1$.

B: Suppose that $a, b \in \mathbb{Z}$ and that $m \in \mathbb{N}$. Prove the following statement:

$$a \equiv b \pmod{m} \implies \gcd(a, m) = \gcd(b, m).$$

Is the converse true? Justify your answer.

C: Find the remainder when $2^{1758421311} - 3^{600}$ is divided by 7.

D: Suppose that $n$ and $m$ are integers and that $m \geq 1$. Consider the following statement:

$$n^2 \equiv 1 \pmod{m} \implies n \equiv 1 \pmod{m} \text{ or } n \equiv -1 \pmod{m}$$

Prove that this statement is true if $m$ is assumed to be a prime. Prove that this statement is false if $m = 8$. 