MATH 310 - Homework Assignment 5 (due Friday, March 2nd))

This homework assignment is based on chapters 11 (section 11.3), chapters 15, 16, 17, and the class lectures. Additional problems will be added to this assignment.

A: Find \( \gcd(a, b) \) for the following pairs of integers:

(i) \( a = 88, \ b = 136 \),  
(ii) \( a = 87, \ b = 135 \),  
(iii) \( a = 7684, \ b = 4148 \),  
(iv) \( a = 7654, \ b = -4148 \),  
(v) \( a = 7684, \ b = 4147 \),  
(vi) \( a = 0, \ b = 17 \)

B: It turns out that \( 87\mathbb{Z} + 135\mathbb{Z} = e\mathbb{Z} \), where \( e \) is a positive integer. Determine \( e \). Find integers \( s, t \) such that \( 87s + 135t = e \).

C: Suppose that \( a \) and \( b \) are nonzero integers. Let \( d = \gcd(a, b) \). Give a careful proof that

\[
\gcd\left( \frac{a}{d}, \frac{b}{d} \right) = 1
\]

D: Suppose that \( a, b, \) and \( c \) are nonzero integers. Prove or disprove the following statement:

\[
\gcd(a, b) \cdot \gcd(a, c) = \gcd(a^2, bc).
\]

(If true, give a proof. If false, give a counterexample.)

E: What are the possible remainders that can result when the square of an integer is divided by 5? What are the possible remainders that result when the fourth power of an integer is divided by 5? Your answers must be justified.

F: What are the possible remainders when the cube of an integer is divided by 7? What are the possible remainders when the sixth power of an integer is divided by 7? Your answers must be justified.

G: Suppose that \( a, b, c \in \mathbb{Z} \). Prove the following statement:

If \( a^2 + b^2 = c^2 \), then 3 divides \( a \) or 3 divides \( b \).