HINTS FOR THE FOURTH PROBLEM SET

Page 94: 2. Theorem 5-14 (which is theorem 2 on the “More Theorems...” handout) is needed. This will allow you to reduce the problem to one concerning continued fractions. Theorem 1 on the “More Theorems...” handout (or its consequence - problem 2 on the third problem set) should then be useful.

Problems 1, 2, 3, and 4 concern $\sqrt{19}$ and are intended to be done in order. You can use the result of problem 3 in problem 4, for example. You will need the answer to problem 1 for problems 2 and 3. Some of the useful theorems are: theorems 9 and 11 from the “Basic Facts...” handout and theorems 1, 2, 5, and 6 from the “More Theorems...” handout.

Problem 3. Think of using theorem 1 on the “More Theorems...” handout (or problem 2 on the third problem set) for the first part. One has to know the continued fraction expansion of $\sqrt{19}$ to do this problem, especially for the second part. You will also need theorem 2 (from “More Theorems...”) for starting the second part. In addition to somehow using theorem 1 from the “More Theorems...” handout (or problem 2 from the third problem set), you may find theorem 9 on the “Basic Facts...” handout useful.

Problem 4. Suppose that $a^2 - 19b^2 = -1$ where $a$ and $b$ are positive integers. Try to get a contradiction to the result proved in the second part of problem 3. Looking at the solution to problem 3 from the first problem set should be useful. You can consider the equation $x^2 - 19y^2 = -1$ instead of $x^2 - 2y^2 = 1$. 