Math 582: Nonnormal Matrices and Linear Operators

(Winter 2005)

Lectures: MWF 1:30–2:20, room 222 Loew

Professor: Anne Greenbaum, C-434 Padelford, 543-1175

Office Hours: Mon, Wed 3:00–4:00, or by appointment.

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Course materials: Click on "Math 582".

Text: Spectra and Pseudospectra by Trefethen and Embree, available from UW Copy Center.

Reserve list: The following books are on reserve in the Mathematics Research Library.

- 1. Topics in Matrix Analysis by Roger Horn and Charles Johnson (QA188.H664, 1991)
- 2. Perturbation Theory for Linear Operators by Tosio Kato (QA329.2.K37, 1995)

This course will cover concepts beyond spectra that can have relevance to problems involving nonnormal matrices and linear operators. A normal matrix is one with a complete set of orthogonal eigenvectors. A nonnormal matrix may or may not be diagonalizable, but if it is diagonalizable, then its eigenvectors are not orthogonal. For nonnormal matrices, eigenvalues may not describe the behavior of the matrix as well as they do for normal matrices. Other related concepts include the field of values or numerical range, the ϵ -pseudospectrum, and the polynomial numerical hull of a given degree. We will discuss each of these sets and prove the most important theorems about their properties. We will also discuss applications.

We will cover selected chapters in the book *Spectra and Pseudospectra* by Trefethen and Embree, and we will cover certain topics from *Topics in Matrix Analysis* by Horn and Johnson. We will also read some papers from the literature. Following is a list of topics to be covered:

- Review of relevant linear algebra; eigenvalues of matrices, spectra of linear operators.
- The field of values or numerical range; Toeplitz-Hausdorf theorem; applications. (ch. 1 of Horn and Johnson; ch. 17 of text).
- Pseudospectra of matrices and linear operators. (ch. 1 of text)
- The polynomial numerical hull of degree k. (Generalizations of the field of values useful in the study of polynomial functions of a matrix, Lin. Alg. Appl. 347 (2002), pp. 233–249)
- Transient effects and nonnormal dynamics. (ch. 14–18 of text)
- Matrix iterations. (ch. 24–29 of text)
- Computation of pseudospectra. (ch. 39-42 of text)
- Further examples and applications. (ch. 56–58 of text)

There will be weekly homework assignments (with some MATLAB programming) (40 %) and a course project (60 %). There will be **no** final exam. The project can take any of the following forms:

- Explore the relevance of spectra, pseudospectra, fields of values, or polynomial numerical hulls in an application area that you are interested in. Give an oral and/or written presentation of your results.
- Report on one or more of the text chapters that are not covered in class.
- Report on a relevant paper from the literature.

I will be giving more specific suggestions about projects as the quarter progresses.