Assignment 4. Due Friday, Feb. 25.

Reading: Secs. 56 and 57 of T & E.

1. Use your 1-norm pseudospectral code to compute pseudospectra of the decay matrix associated with the Ehrenfest urn / random walk on a hypercube problem, taking, say, $n = 39$. The transition matrix P is given on p. 486 in the text, and recall that the decay matrix is $A = P - P^{\infty}$. Also note that if you use the standard MATLAB 1-norm function (the maximum absolute column sum), then you will need to work with A^T, the transpose of the matrix described in the text. Turn in a plot of computed 1-norm pseudospectra.

   Using estimates that we have discussed in class or that you can find in the text, explain what information the pseudospectra give about transient behavior of this Markov chain.

2. Try to compute the 1-norm polynomial numerical hulls of various degrees for this problem. [One way to do this is to take your code to compute 2-norm polynomial numerical hulls and replace the call to SDPT3 by a call to the MATLAB minimization routine `fminunc`. To minimize $\|I - \sum_{j=1}^{k} c_j (A - \zeta I)^j\|_1$, you will have to supply a function which, given $c_1, \ldots, c_j$ (with, say, the real parts of these coefficients stored in $c(1:k)$ and the imaginary parts in $c(k+1:2*k)$, since, I believe, `fminunc` minimizes over *real* variables), computes the 1-norm of the matrix $I - \sum_{j=1}^{k} c_j (A - \zeta I)^j$. Hopefully, `fminunc` will then vary the coefficients to minimize this norm and you can check at the end to see if it is less than 1 (or less than, say, .9999); if so, then ζ is outside the hull, and otherwise it is inside. Unfortunately, `fminunc` is not guaranteed to find the global minimum, and you may need to try some different initial guesses to be safe. This is the way that I have computed 1-norm polynomial numerical hulls. If you come up with a better way, and if you still have not found a project topic, explaining a better way to do this computation could make an excellent project.]

   If you are successful at computing 1-norm polynomial numerical hulls of some different degrees, explain what information they give about transient behavior of the Markov chain.